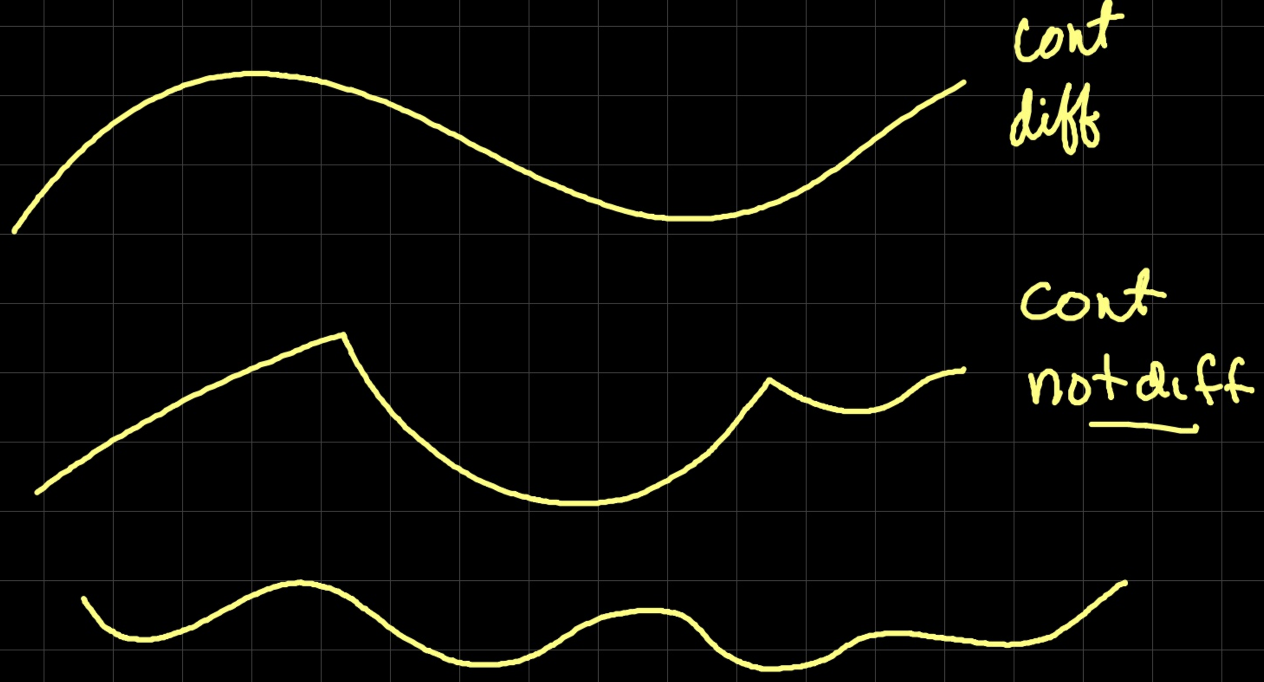
Differentiability



If diff then cont.

If not cont then not diff

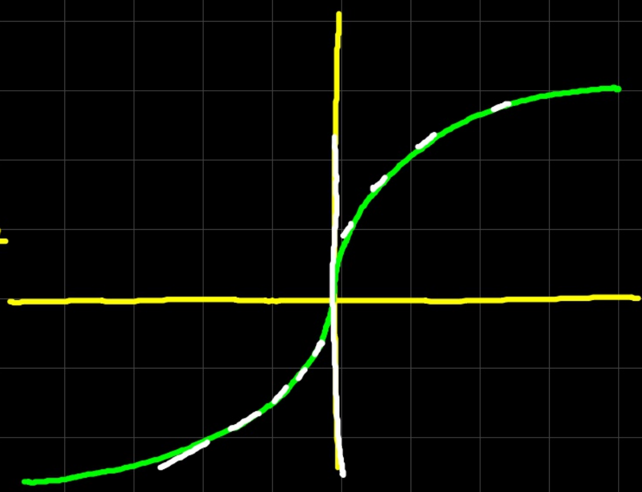
If cont then diff

Vertical Tangent

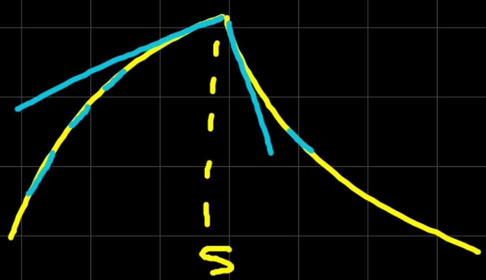
$$f(x) = \sqrt[3]{x}$$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$$

Since $f'(0) \nexists$,
f is not diff.



CORNERS / CUSPS



$f'_+(s) \neq f'_-(s) \longrightarrow f'(s) \nexists$

$$f(x) = |x-2| = \begin{cases} x-2 & x \geq 2 \\ 2-x & x < 2 \end{cases}$$

$$f'(x) = \begin{cases} 1 & x > 2 \\ -1 & x < 2 \end{cases}$$

$f'_+(2) = 1$ but

$f'_-(2) = -1 \rightarrow f'(2) \nexists$

$\therefore$ not diff

at $x = 2$.

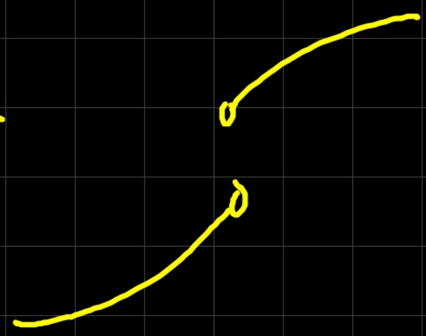
Continuity Issues

$f(a) \nexists$

$\pm\infty$

$\lim\limits_{x \to a} f(x) \nexists$

$L \neq R$



$$f(x) = \begin{cases} x^3 - 3x & x \le 1 \\ 5x & x > 1 \end{cases}$$

$$\begin{array}{cc} 3x^2 - 3 & 0 \\ 5 & 5 \end{array}$$

$$f'(x) = \begin{cases} 3x^2 - 3 & x < 1 \\ 5 & x > 1 \end{cases}$$

$f'_+(1) = 5$ but $f'_-(1) = 0 \longrightarrow f'(1) \nexists$

$\therefore f$ not diff at $x = 1$.

$2x \quad 4$
$2x \quad 4$

$$f(x) = \begin{cases} x^2+3 & x < 2 \\ x^2-4 & x \geq 2 \end{cases}$$

Cont test at $x=2$

$f(2) = 0$

$\lim_{x \to 2^+} f(x) = 0$
$\lim_{x \to 2^-} f(x) = 7$
$\therefore \lim_{x \to 2} f(x)\ \nexists$

Since $\lim_{x \to 2} f(x)\ \nexists$ f is not cont at $x=2$

$\therefore$ f is not diff. at $x=2$.

$$f(x)=\begin{cases} x^2+1 & x\le 1 \\ 2x & x>1 \end{cases}$$

$2x \to 2$
$2 \to 2$

Cont test at $x=1$

$f(1)=2$

$$\left.\begin{array}{l} \lim\limits_{x\to 1^+} f(x)=2 \\ \lim\limits_{x\to 1^-} f(x)=2 \end{array}\right\} \therefore \lim_{x\to 1} f(x)=2$$

f is cont at $x=1$

Deriv

$$f'(x)=\begin{cases} 2x & x<1 \\ 2 & x>1 \end{cases}$$

Since $f'_+(1)=2$ and $f'_-(1)=2 \to f'(1)=2$

and f is cont at $x=1$

$\therefore f$ is diff at $x=1$

$$f(x) = \begin{cases} 5x-3 & x \le -2 \\ x^2 & -2 < x < 3 \\ x^3 & x \ge 3 \end{cases}$$

$5 \to 5$

$2x \to -4 \quad 6$

$3x^2 \to \quad 27$

$$f'(x) = \begin{cases} 5 & x < -2 \\ 2x & -2 < x < 3 \\ 3x^2 & x > 3 \end{cases}$$

$f'_+(-2) = -4$ but $f'_-(-2) = 5 \to f'(-2)\nexists$

$\therefore f$ not diff at $x = -2$

$f'_+(3) = 27$ but $f'_-(3) = 6 \to f'(3)\nexists$

$\therefore f$ not diff at $x = 3$.

$$f(x)=\begin{cases} x^2 & x\le 1 \\ ax+7 & x>1 \end{cases}$$

find a so that f is diff $\forall x$.

$$f'(x)=\begin{cases} 2x & x<1 \\ a & x>1 \end{cases}$$

$f'_+(1)=a$ and $f'_-(1)=2$

$\therefore a=2.$

$$f(x) = \begin{cases} x^2 & x \le 1 \\ ax+b & x > 1 \end{cases}$$ find a & b so that f is diff $\forall x$.

<u>Cont test at $x=1$</u>

$f(1) = 1$

$\lim_{x \to 1^+} f(x) = a+b$

$\lim_{x \to 1^-} f(x) = 1$

for f to be cont,

$a+b=1$

<u>Deriv</u>

$$f'(x) = \begin{cases} 2x & x < 1 \\ a & x > 1 \end{cases}$$

$f'_+(1) = a$ and $f'_-(1) = 2$

$\therefore a = 2$

$\therefore$ for f to be diff $\forall x$,

$a=2$ and $b=-1$.