The Chain Rule

Let $h(x) = f(g(x))$

$$h'(x) = \lim_{x_1 \to x} \frac{h(x_1) - h(x)}{x_1 - x_1}$$

$$= \lim_{x_1 \to x} \frac{f(g(x_1)) - f(g(x))}{x_1 - x} \cdot \frac{g(x_1) - g(x)}{g(x_1) - g(x)}$$

$$= \lim_{x_1 \to x} \frac{f(g(x_1)) - f(g(x))}{g(x_1) - g(x)} \cdot \frac{g(x_1) - g(x)}{x_1 - x}$$

$$= \boxed{\lim_{g(x_1) \to g(x)} \frac{f(g(x_1)) - f(g(x))}{g(x_1) - g(x)}} \cdot \boxed{\lim_{x_1 \to x} \frac{g(x_1) - g(x)}{x_1 - x}}$$

$$= f'(g(x))\, g'(x)$$

THE CHAIN RULE

If $h(x) = f(g(x)) \longrightarrow h'(x) = f'(g(x))\,g'(x)$

$$f(x) = (x^2 + 5x - 2)^{10}$$

$$f'(x) = 10\,(x^2 + 5x - 2)^9\,(2x + 5)$$

$$g(x) = \sin 8x$$

$$g'(x) = 8\cos 8x$$

$$D_x\left[f(x)^n\right] = n\, f(x)^{n-1} f'(x)$$ Power Rule

$$D_x\left[\sqrt{f(x)}\right] = \frac{f'(x)}{2\sqrt{f(x)}}$$

$$f(x) = \sin x^3$$
$$f'(x) = 3x^2 \cos x^3$$

$$D_x\left[\sin f(x)\right] = f'(x) \cos f(x)$$

$$D_x\left[\cos f(x)\right] = -f'(x) \sin f(x)$$

$$D_x\left[\tan f(x)\right] = f'(x) \sec^2 f(x)$$

$$D_x\left[\sec f(x)\right] = f'(x) \sec f(x) \tan f(x)$$

$$D_x\left[\cot f(x)\right] = -f'(x) \csc^2 f(x)$$

$$D_x\left[\csc f(x)\right] = -f'(x) \csc f(x) \cot f(x)$$

$$f(x) = x^3 \sin 4x$$

$$f'(x) = (x^3)(4\cos 4x) + (\sin 4x)(3x^2)$$

$$f(x) = \sec x^2$$

$$f'(x) = 2x \sec x^2 \tan x^2$$

$$g(x) = \frac{5}{x^4 + x^3 - 6} = 5(x^4 + x^3 - 6)^{-1}$$

$$g'(x) = -5(x^4 + x^3 - 6)^{-2}(4x^3 + 3x^2)$$

$$= -\frac{5(4x^3 + 3x^2)}{(x^4 + x^3 - 6)^2}$$

Another Version of Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Given $y = u^3$ and $u = 5x - 6$ find $\frac{dy}{dx}$.

$\frac{dy}{du} = 3u^2 \qquad \frac{du}{dx} = 5$

$$\frac{dy}{dx} = 3u^2(5)$$

$$= 3(5x-6)^2(5)$$

$y = (5x-6)^3$

$$\frac{dy}{dx} = 3(5x-6)^2(5)$$

$$f'(g(x))\, g'(x)$$

$$f(x) = g(x^3)$$

$$f'(x) = g'(x^2) \cdot 2x$$

$$h(x) = \sec^3(\sin x)$$

$$h'(x) = \left[3\sec^2(\sin x)\right]\left[\sec(\sin x)\tan(\sin x)\right]\left[\cos x\right]$$

$$g(x) = \sqrt{\tan(\sin x)}$$

$$g'(x) = \frac{\left[\sec^2(\sin x)\right]\cos x}{2\sqrt{\tan(\sin x)}}$$

$$f(x) = \sqrt{x^2+4x}$$

$$f'(x) = \frac{2x+4}{2\sqrt{x^2+4}}$$

$$f(x) = \sqrt[4]{x^2+4x}$$

$$f'(x) = \frac{2x+4}{4\sqrt[4]{(x^2+4x)^3}}$$

$$f(x) = \sqrt[3]{x^2+4x}$$

$$f'(x) = \frac{2x+4x}{3\sqrt[3]{(x^2+4x)^2}}$$

$$f(x) = \sqrt[6]{\sin x}$$

$$f'(x) = \frac{\cos x}{6\sqrt[6]{\sin^5 x}}$$

$$D_x\left[\sqrt[n]{f(x)}\right] = \frac{f'(x)}{n\sqrt[n]{f(x)^{n-1}}}$$