Differentiation Theorems

$$D_x[c] = 0$$

$$D_x[x^n] = n x^{n-1} \quad \text{Power Rule}$$

$$D_x[\sqrt{x}] = \frac{1}{2\sqrt{x}}$$

$$x^{23} \rightarrow 23x^{22}$$

$$x^{2/3} \rightarrow \frac{2}{3}x^{-1/3}$$

$$8x^3 \rightarrow 24x^2$$

$$D_x\left[f(x)\,g(x)\right] = f(x)\,g'(x) + g(x)\,f'(x).$$

Product Rule

$6x^2 + x - 12$

$f(x) = (3x-4)(2x+3)$

$f'(x) = (3x-4)(2) + (2x+3)(3)$

$= 6x - 8 + 6x + 9$

$= 12x + 1$

$12x + 1$

~~$f(x) = x^3 \sin x$~~

Quotient Rule

$$D_x\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$f(x) = \frac{2x+3}{5x-1}$$

$$f'(x) = \frac{(5x-1)(2) - (2x+3)(5)}{(5x-1)^2}$$

$$= \frac{10x - 2 - 10x - 15}{(5x-1)^2}$$

$$= -\frac{17}{(5x-1)^2}$$

$$f(x) = x^{23} - 30x^3 + 3$$

$$f'(x) = 23x^{22} - 90x^2$$

$$h(x) = (x^2+1)(2x-3)$$

$$h'(x) = (x^2+1)(2) + (2x-3)(2x)$$

$$= 2x^2 + 2 + 4x^2 - 6x$$

$$= 6x^2 - 6x + 2$$

$$R(y) = \frac{10}{\sqrt{y}}$$

$$= 10y^{-1/2}$$

$$R'(y) = -5y^{-3/2}$$

$$= -\frac{5}{\sqrt{y^3}}$$

$$R'(y) = \frac{(\sqrt{y})(0) - (10)\frac{1}{2\sqrt{y}}}{y}$$

$$= \frac{-5}{y\sqrt{y}}$$

$$u = v^4 - \sqrt[4]{v}$$

$$= v^4 - v^{1/4}$$

$$\frac{du}{dv} = 4v^3 - \frac{1}{4}v^{-3/4}$$

$$= 4v^3 - \frac{1}{4\sqrt[4]{v^3}}$$

$$s(t) = \sqrt{t}\left(t^3 - \sqrt{t} + 1\right)$$

$$s'(t) = (\sqrt{t})\left(3t^2 - \frac{1}{2\sqrt{t}}\right) + \left(t^3 - \sqrt{t} + 1\right)\frac{1}{2\sqrt{t}}$$

$$g(x) = \frac{1}{x^3} + \frac{2}{x^2} + \frac{3}{\sqrt{x}}$$

$$= x^{-3} + 2x^{-2} + 3x^{-1/2}$$

$$g'(x) = -3x^{-4} - 4x^{-3} - \frac{3}{2}x^{-3/2}$$

$$= -\frac{3}{x^4} - \frac{4}{x^3} - \frac{3}{2\sqrt{x^3}}$$

WRITING EQUATIONS OF TANGENTS

I. Write eq tn (normal) to $f(x) = \frac{10}{14-x^2}$ at $x = 4$.

$$f(4) = \frac{10}{-2} = -5 \longrightarrow (4, -5)$$

$$f'(x) = \frac{(14-x^2)(0) - (10)(-2x)}{(14-x^2)^2}$$

$$f'(4) = \frac{80}{4} = 20 \longrightarrow m_T = 20 \quad \left(m_\perp = -\frac{1}{20}\right)$$

$$\therefore y + 5 = 20(x-4)$$

$$-\frac{1}{20}$$

II Write eq tan to $y = 3x^2 - 4x$ that is parallel to $2x - y + 3 = 0$.

perpendi.

Slope of given line $m = 2$ $\therefore m_\perp = -\frac{1}{2}$

$\frac{dy}{dx} = 6x - 4$ $\quad -\frac{1}{2}$

$\therefore 6x - 4 = 2 \longrightarrow x = 1 \longrightarrow (1, -1)$

$$\therefore y + 1 = 2(x - 1)$$

III Write eq tan to $y = x^2$ that passes thru $(5,9)$.

Slope of line thru $(5,9)$ and (x, x^2) is $\frac{x^2-9}{x-5}$.

$$\frac{dy}{dx} = 2x$$

$$\therefore 2x = \frac{x^2-9}{x-5} \longrightarrow x = 1 \wedge x = 9$$

$$\therefore (1,1) \text{ and } (9,81)$$

At $(1,1)$ $\left.\frac{dy}{dx}\right|_{x=1} = 2 \rightarrow m_T = 2$ $\therefore y - 1 = 2(x-1)$

At $(9,81)$ $\left.\frac{dy}{dx}\right|_{x=9} = 18 \rightarrow m_T = 18$ $\therefore y - 81 = 18(x-9)$