This exercise is not intended as a comprehensive
review of the material.

It is a review...but the main purpose is to practice
our presentation skills.

Limits Test--14 problems--70 total points

7 limit problems
1 HA
1 VA
1 IVT
1 limits from a graph

1 limit proof
1 continuity
1 remove and redfine

1. Evaluate: $\lim_{x \to 3} \frac{x^2 + 4x - 21}{x - 3}$.

$$\lim_{x \to 3} \frac{\overset{1}{\cancel{(x-3)}}(x+7)}{\cancel{x-3}} = 10$$

2. Evaluate: $\lim_{x \to 0} \frac{\sqrt{4-x}-2}{x}$.

$$\lim_{x \to 0} \frac{\sqrt{4-x}-2}{x} \cdot \frac{\sqrt{4-x}+2}{\sqrt{4-x}+2}$$

$$= \lim_{x \to 0} \frac{\cancel{4} \cancel{-x}^{-1} \cancel{-4}}{\cancel{x}(\sqrt{4-x}+2)}$$

$$= -\frac{1}{4}$$

3. Evaluate: $\lim_{x \to 3} \frac{x-6}{x-3}$.

$$\lim_{x \to 3} \frac{x-6}{x-3} \nexists$$

$$\lim_{x \to 3^+} \frac{x-6}{x-3} = -\infty \quad \text{and} \quad \lim_{x \to 3^-} \frac{x-6}{x-3} = +\infty$$

4. Evaluate: $\lim_{x\to\infty} \dfrac{5-2x-7x^3}{2x^3-11x}$.

$$\lim_{x\to\infty} \frac{5-2x-7x^3}{2x^3-11x} = -\frac{7}{2}.$$

5. Evaluate: $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$.

$$\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25} \cdot \frac{\sqrt{x} + 5}{\sqrt{x} + 5}$$

$$= \lim_{x \to 25} \frac{\cancel{x - 25}^{\,1}}{\cancel{(x - 25)}(\sqrt{x} + 5)}$$

$$= \frac{1}{10}$$

6. Evaluate: $\lim_{x\to 0} \frac{\sin 5x}{\sin 2x}$.

$$\lim_{x \to 0} \frac{\sin 5x}{\sin 2x} = \frac{5}{2}.$$

OR

$$\lim_{x \to 0} \frac{\frac{\sin 5x}{5x} \cdot 5\cancel{x}}{\frac{\sin 2x}{2x} \cdot 2\cancel{x}} = \frac{5}{2}$$

7. Evaluate: $\lim_{x \to -\infty} \frac{\sqrt{5x^2 - 2x}}{3x+6}$.

$$\lim_{x \to -\infty} \frac{\sqrt{5x^2-2x}}{3x+6} = -\frac{\sqrt{5}}{3}.$$

8. Find the vertical asymptotes (if any) of $f(x) = \dfrac{x+5}{(x-2)^2}$.

f has a VA at $x = 2$ because

$f(2) \nexists$ and $\lim_{x \to 2} f(x) = \pm\infty$.

9. Find the horizontal asymptotes (if any) of $g(x) = \dfrac{5x-1}{\sqrt{3x^2+6}}$.

g has a HA at $y = \frac{5}{\sqrt{3}}$ because $\lim_{x\to\infty} g(x) = \frac{5}{\sqrt{3}}$.

g has a HA at $y = -\frac{5}{\sqrt{3}}$ because $\lim_{x\to-\infty} g(x) = -\frac{5}{\sqrt{3}}$.

10. The function $f(x) = \dfrac{2x^2+5x-7}{x-1}$ is discontinuous at $x = 1$. Determine if the discontinuity is removable. If removable, redefine f so that it is continuous at $x = 1$.

Removable?

The discon. is removable because $\lim_{x \to 1} f(x) = 9$.

Redefine f

$$f(x) = \begin{cases} \dfrac{2x^2+5x-7}{x-1} & x \neq 1 \\ 9 & x = 1 \end{cases}$$

11. Determine where the following function is discontinuous: $f(x) = \begin{cases} x^2 - 4 & \text{if } x \leq -2 \\ 4 - x^2 & \text{if } -2 < x < 2 \\ 3 + x & \text{if } x \geq 2 \end{cases}$.

Cont test at $x=-2$

$f(-2)=0$

$\left.\begin{array}{l} \lim_{x \to -2^+} f(x) = 0 \\ \lim_{x \to -2^-} f(x) = 0 \end{array}\right\} \therefore \lim_{x \to -2} f(x) = 0$

$\therefore$ f is cont at $x=-2$

because $f(-2) = \lim_{x \to -2} f(x)$.

Cont test at $x=2$

$f(2)=5$

$\left.\begin{array}{l} \lim_{x \to 2^+} f(x) = 5 \\ \lim_{x \to 2^-} f(x) = 0 \end{array}\right\} \therefore \lim_{x \to 2} f(x) \nexists$

$\therefore$ f is discon because

$\lim_{x \to 2} f(x) \nexists$.

12. Given $f(x) = x^3 - 4x^2 + 4x + 3$, use the Intermediate Value Theorem to show that $f(x) = 0$ for some x between $x = -1$ and $x = 2$.

Since $f(-1) = -6 < 0$ and $f(2) = 3 > 0$

then by IVT $f(x) = 0$ for some $x \in (-1, 2)$.

13. Prove: $\lim_{x \to 2} x^2 + 2x + 1 = 9$

We need to show that $\forall \varepsilon > 0 \ \exists \delta > 0 \ \ni$ when $|x-2| < \delta \rightarrow |(x^2+2x+1)-9| < \varepsilon$

$$|x^2+2x-8| < \varepsilon$$

$$|x-2||x+4| < \varepsilon$$

$$|x-2| < \frac{\varepsilon}{|x+4|}$$

Consider $[1, 3]$

$$x=1 \rightarrow \frac{\varepsilon}{|x+4|} = \frac{\varepsilon}{5}$$

$$x=3 \rightarrow \frac{\varepsilon}{|x+4|} = \frac{\varepsilon}{7}$$

$\therefore$ Choose $\delta = \min\left\{1, \frac{\varepsilon}{7}\right\}$.

[redacted]