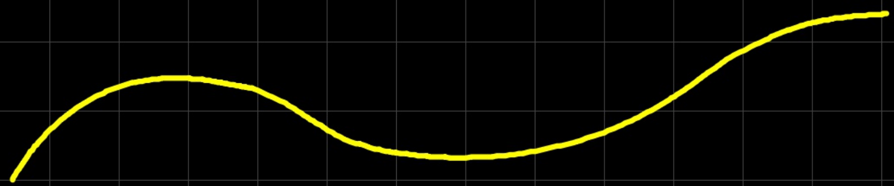
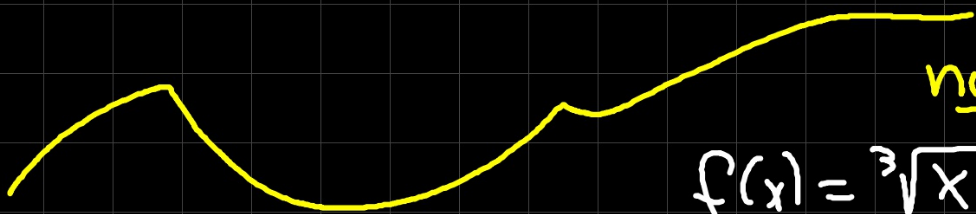
Differentiability



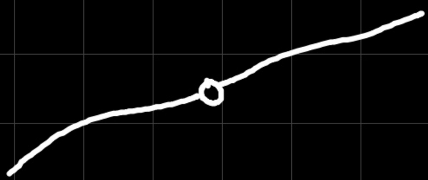
cont
differentiable



cont
<u>not</u> diff

$$f(x) = \sqrt[3]{x}$$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$$



Diff $\to$ Cont Always

$\sim$ Cont $\to$ $\sim$ Diff Always

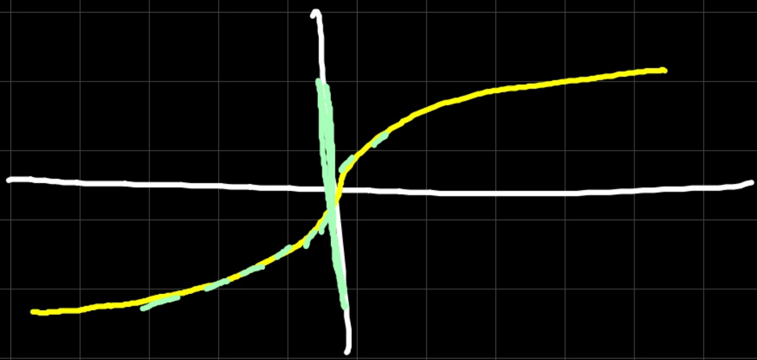
Cont $\to$ Diff sometimes

VERTICAL TANGENTS

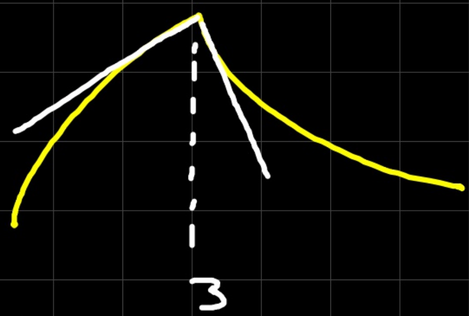
$$f(x) = \sqrt[3]{x}$$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$$

$f'(0) \nexists$ but $f(0) \exists \rightarrow$ Vertical tan



Corners / Cusps



$f'_+(3) \neq f'_-(3) \longrightarrow f'(3) \nexists$

$$f(x) = |x-2| = \begin{cases} x-2 & x \geq 2 \\ 2-x & x < 2 \end{cases}$$

$$f'(x) = \begin{cases} 1 & x > 2 \\ -1 & x < 2 \end{cases}$$

$f'_+(2) = 1$ but

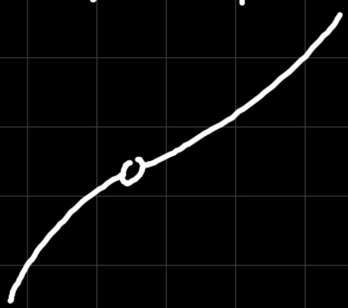
$f'_-(2) = -1 \longrightarrow f'(2) \nexists$

$\therefore f$ not diff

at $x = 2$.

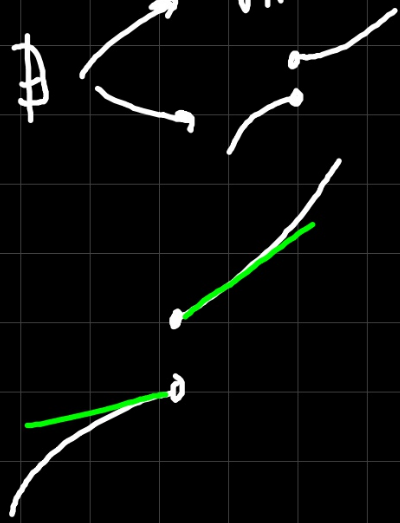
CONTINUITY ISSUE

$f(a)\ \nexists$



$\lim\limits_{x \to a} f(x)\ \nexists$

VA



$$3x^2-3 \quad 0$$
$$5 \quad 5$$

$$f(x)=\begin{cases} x^3-3x & x\le 1 \\ 5x & x>1 \end{cases}$$

$$f'(x)=\begin{cases} 3x^2-3 & x<1 \\ 5 & x>1 \end{cases}$$

$f'_+(1)=5$ but $f'_-(1)=0 \longrightarrow f'(1) \nexists$

$\therefore$ f not diff. at $x=1$.

$2x \to 4$
$2x \to 4$

$$f(x) = \begin{cases} x^2+3 & x \le 2 \\ x^2-4 & x > 2 \end{cases}$$

Cont test at $x=2$

$f(2) = 7$

$$\left.\begin{array}{l} \lim\limits_{x\to 2^-} f(x) = 7 \\ \lim\limits_{x\to 2^+} f(x) = 0 \end{array}\right\} \therefore \lim_{x\to 2} f(x) \nexists$$

f is disc. at $x=2$ because
$\lim\limits_{x\to 2} f(x) \nexists$ $\therefore$ f is not diff at $x=2$.

$$f(x) = \begin{cases} x^2+1 & x \le 1 \\ 2x & x > 1 \end{cases}$$

$2x \to 2$

$2 \to 2$

Cont test at $x=1$

$f(1) = 2$

$$\left.\begin{array}{l} \lim_{x \to 1^+} f(x) = 2 \\ \lim_{x \to 1^-} f(x) = 2 \end{array}\right\} \therefore \lim_{x \to 1} f(x) = 2$$

Since $f(1) = \lim_{x \to 1} f(x)$,

f is cont at $x=1$.

Deriv

$$f'(x) = \begin{cases} 2x & x < 1 \\ 2 & x > 1 \end{cases}$$

$f'_+(1) = 2$ and $f'_-(1) = 2$

$\to f'(1) = 2$

and f is cont at $x=1$

$\therefore$ f is diff at $x=1$.

$$f(x) = \begin{cases} x^2+2 & x \le 1 \\ x+2 & x > 1 \end{cases}$$

$2x \to 2$
$1 \to 1$

$$f'(x) = \begin{cases} 2x & x < 1 \\ 1 & x > 1 \end{cases}$$

$f'_+(1) = 1$ but $f'_-(1) = 2 \longrightarrow f'(1) \nexists$

$\therefore f$ not diff at $x=1$.

$$f(x) = \begin{cases} 5x-2 & x \le -2 \\ x^2 & -2 < x < 3 \\ x^3 & x \ge 3 \end{cases}$$

$5 \to 5$

$2x \to -4 \quad 6$

$3x^2 \longrightarrow 27$

$$f'(x) = \begin{cases} 5 & x < -2 \\ 2x & -2 < x < 3 \\ 3x^2 & x > 3 \end{cases}$$

$f'_+(-2) = -4$ but $f'_-(-2) = 5 \longrightarrow f'(-2) \nexists$

$\therefore$ f not diff at $x = -2$.

$f'_+(3) = 27$ but $f'_-(3) = 6 \longrightarrow f'(3) \nexists$

$\therefore$ f not diff at $x = 3$.

$$f(x) = \begin{cases} x^2 & x < 1 \\ ax+b & x \geq 1 \end{cases}$$

find a & b so that f is diff $\forall x$.

<u>Cont test at $x=1$</u>

$f(1) = a+b$

$\lim_{x \to 1^-} f(x) = 1$

$\lim_{x \to 1^+} f(x) = a+b$

for f to be cont,

$a+b=1$

<u>Deriv</u>

$$f'(x) = \begin{cases} 2x & x < 1 \\ a & x > 1 \end{cases}$$

$f'_+(1) = a$ and $f'_-(1) = 2$

$\therefore a = 2$

$\therefore$ for f to be diff $\forall x$

$a = 2$ and $b = -1$