# The Chain Rule

Let $h(x) = f(g(x))$

$$h'(x) = \lim_{x_1 \to x} \frac{h(x_1) - h(x)}{x_1 - x}$$

$$= \lim_{x_1 \to x} \frac{f(g(x_1)) - f(g(x))}{x_1 - x} \cdot \frac{g(x_1) - g(x)}{g(x_1) - g(x)}$$

$$= \lim_{g(x_1) \to g(x)} \frac{f(g(x_1)) - f(g(x))}{g(x_1) - g(x)} \cdot \frac{g(x_1) - g(x)}{x_1 - x}$$

$$= \boxed{\lim_{g(x_1) \to g(x)} \frac{f(g(x_1)) - f(g(x))}{g(x_1) - g(x)}} \cdot \boxed{\lim_{x_1 \to x} \frac{g(x_1) - g(x)}{x_1 - x}}$$

$$= f'(g(x))\, g'(x)$$

# THE CHAIN RULE

If $h(x) = f(g(x)) \longrightarrow h'(x) = f'(g(x))\,g'(x)$

$f(x) = (5x-3)^8$

$f'(x) = 8(5x-3)^7(5)$

$g(x) = \sin^3 x$

$g'(x) = 3\sin^2 x \cos x$

$$D_x\left[f(x)^n\right] = n\, f(x)^{n-1} f'(x)$$

$$D_x\left[\sqrt{f(x)}\right] = \frac{f'(x)}{2\sqrt{f(x)}}$$

$$D_x\left[\sin f(x)\right] = f'(x) \cos f(x)$$

$$D_x\left[\cos f(x)\right] = -f'(x) \sin f(x)$$

$$D_x\left[\tan f(x)\right] = f'(x) \sec^2 f(x)$$

$$D_x\left[\cot f(x)\right] = -f'(x) \csc^2 f(x)$$

$$D_x\left[\sec f(x)\right] = f'(x) \sec f(x) \tan f(x)$$

$$D_x\left[\csc f(x)\right] = -f'(x) \csc f(x) \cot f(x)$$

$$h'(x) = \underbrace{f'(g(x))}\,\underbrace{g'(x)}$$

# ANOTHER VERSION OF CHAIN RULE

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Given $y = u^3$ and $u = 5x+3$ find $\frac{dy}{dx}$.

$$\frac{dy}{du} = 3u^2 \qquad \frac{du}{dx} = 5$$

$$\frac{dy}{dx} = 3u^2 \cdot 5$$

$$= 3(5x+3)^2 \cdot 5$$

$$y = (5x+3)^3$$

$$\frac{dy}{dx} = 3(5x+3)^2 5$$

$$f(x) = (x^2+3x-6)^{10}$$
$$f'(x) = 10(x^2+3x-6)^9(2x+3)$$

$$g(x) = x^2\sin^4 x$$
$$g'(x) = (x^2)(4\sin^3 x\cos x) + (\sin^4 x)(2x)$$

$$g(x) = \frac{5}{x^3-7x+3} = 5(x^3-7x+3)^{-1}$$
$$g'(x) = -5(x^3-7x+3)^{-2}(3x^2-7)$$
$$= -\frac{5(3x^2-7)}{(x^3-7x+3)^2}$$

$$f(x) = \sin(\cos x^3)$$

$$f'(x) = \left[\cos(\cos x^3)\right]\left[-\sin x^3\right]\left[3x^2\right]$$

$$f(x) = \sqrt{\sec x^2}$$

$$f'(x) = \frac{2x \sec x^2 \tan x^2}{2\sqrt{\sec x^2}}$$

$$\sqrt{f(x)} \longrightarrow \frac{f'(x)}{2\sqrt{f(x)}}$$

$$D_x\left[\sqrt[n]{f(x)}\right] = \frac{f'(x)}{n\sqrt[n]{f(x)^{n-1}}}$$

$$h(x) = \sqrt[4]{\sin 5x}$$

$$h'(x) = \frac{5\cos 5x}{4\sqrt[4]{\sin^3 5x}}$$

$$f(x) = \sqrt{1+\sqrt{x}}$$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}}{2\sqrt{1+\sqrt{x}}}$$

$$h(x) = \sec(\underline{\sec(\underline{\sin x})})$$

$$h'(x) = \sec(\underline{\sec}(\sin x))\tan(\underline{\sec(\sin x)})\sec(\sin x)\tan(\sin x)\cos x$$