Differentiation Theorems

$$D_x[c] = 0$$

$$D_x[x^n] = nx^{n-1} \longrightarrow \text{Power Rule}$$

$$D_x[x^3] = 3x^2$$

$$D_x[\sqrt{x}] = \frac{1}{2\sqrt{x}}$$

$$D_x[5x^4] = 20x^3$$

$$f(x) = \sqrt[3]{x^5} = x^{5/3}$$

$$f'(x) = \frac{5}{3}x^{2/3} = \frac{5}{3}\sqrt[3]{x^2}$$

Product Rule

$$D_x\left[f(x)g(x)\right] = f(x)g'(x) + g(x)f'(x)$$

$f(x) = (3x+1)(x-5)$ $= 3x^2 - 14x - 5$

$f'(x) = (3x+1)(1) + (x-5)(3)$ $f'(x) = 6x - 14$

$= 3x + 1 + 3x - 15$

$= 6x - 14$

$g(x) = x^2 \sin x$

Quotient Rule

$$D_x\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{(g(x))^2}$$

$$h(x) = \frac{5x+3}{2x-7}$$

$$h'(x) = \frac{(2x-7)(5)-(5x+3)(2)}{(2x-7)^2}$$

$$= \frac{10x-35-10x-6}{(2x-7)^2}$$

$$= -\frac{41}{(2x-7)^2}$$

$$g(x) = \frac{x^2 - 5x + 1}{\sqrt{x}}$$

$$g'(x) = \frac{(\sqrt{x})(2x-5) - (x^2 - 5x + 1)\frac{1}{2\sqrt{x}}}{x}$$

$$h(x) = x^{23} - 30x^2 + 3$$

$$h'(x) = 23x^{22} - 60x$$

$$h(x) = (x^2+1)(2x-7)$$

$$h'(x) = (x^2+1)(2) + (2x-7)(2x)$$

$$f(x) = \frac{10}{\sqrt{x}} = 10x^{-1/2}$$

$$f'(x) = -5x^{-3/2}$$

$$= -\frac{5}{\sqrt{x^3}}$$

$$t(w) = \frac{\sqrt{5}}{w^5} = \sqrt{5}\, w^{-5}$$

$$t'(w) = -5\sqrt{5}\, w^{-6}$$

$$= - \frac{5\sqrt{5}}{w^6}$$

$$y = \frac{1-x^2}{1+x^2}$$

$$\frac{dy}{dx} = \frac{(1+x^2)(-2x) - (1-x^2)(2x)}{(1+x^2)^2}$$

$$u = v^4 - \sqrt[4]{v}$$

$$= v^4 - v^{1/4}$$

$$\frac{du}{dv} = 4v^3 - \frac{1}{4} v^{-3/4}$$

$$= 4v^3 - \frac{1}{4\sqrt[4]{v^3}}$$

EQUATIONS OF TANGENTS TO CURVES

I. Write an eq of a tan to $f(x)=\frac{10}{14-x^2}$ at $x=4$.

$$f(4)=-5 \rightarrow (4,-5)$$

$$f'(x)=\frac{(14-x^2)(0)-(10)(-2x)}{(14-x^2)^2}$$

$$f'(4)=\frac{80}{4}=20 \longrightarrow m_T=20$$

$$\therefore\ y+5=20(x-4)$$

II Write an eq of tan to $y = 3x^2 - 4x$

that is ~~parallel~~ to $2x - y + 3 = 0$.

perpendicular

Slope of given line: $m = 2 \rightarrow m_{\perp} = -\frac{1}{2}$

$$\frac{dy}{dx} = 6x - 4 = -\frac{1}{2}$$

$\therefore 6x - 4 = 2 \rightarrow x = 1 \rightarrow (1, -1)$

$\therefore y + 1 = 2(x - 1)$

III Write eq tan to $y = x^2$ that passes thru $(5, 9)$.

Slope of line thru $(5,9)$ & (x, x^2) is $\frac{x^2 - 9}{x - 5}$.

$$\frac{dy}{dx} = 2x$$

$$\therefore 2x = \frac{x^2 - 9}{x - 5} \longrightarrow \begin{array}{lcl} x = 1 & \text{or} & x = 9 \\ y = 1 & & y = 81 \end{array}$$

At $(1,1) \rightarrow \left.\frac{dy}{dx}\right|_{x=1} = 2 \quad \therefore y - 1 = 2(x-1)$

At $(5,9) \rightarrow \left.\frac{dy}{dx}\right|_{x=9} = 18 \quad \therefore y - 81 = 18(x-9)$