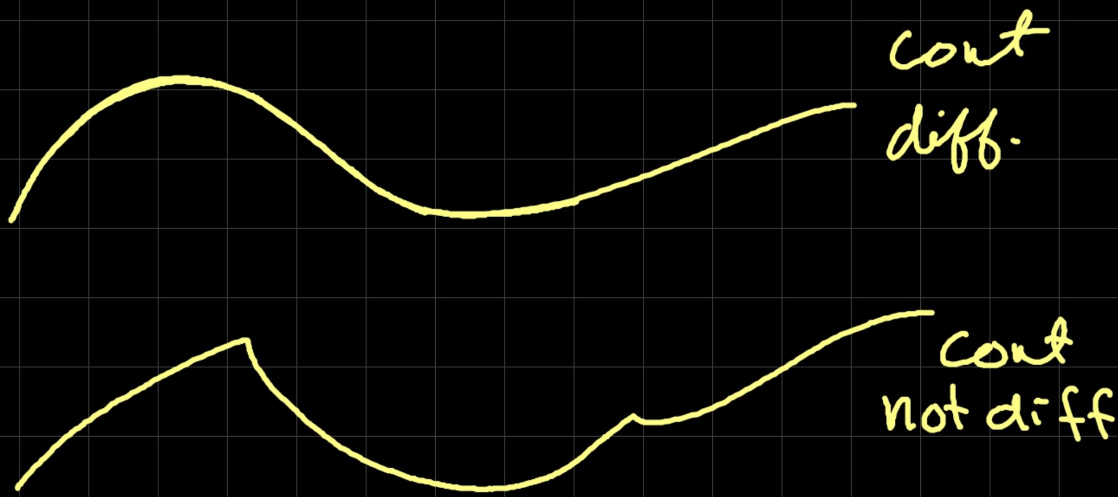
Differentiability

$$f(x) = \sqrt[3]{x}$$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$$

Since $f'(0) \nexists$ f is not diff. at $x = 0$.



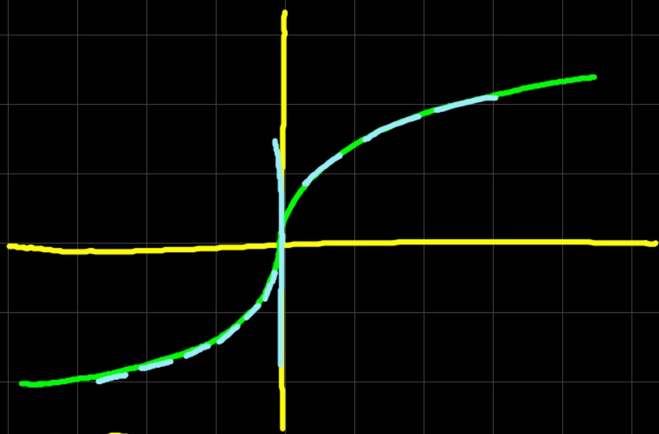
Diff → Cont

Vertical Tangent

$$f(x) = \sqrt[3]{x}$$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$$

$f'(0) \nexists$ but $f(0) \exists$



CORNERS / CUSPS

$$f(x) = |x-2| = \begin{cases} x-2 & x \geq 2 \\ 2-x & x < 2 \end{cases}$$

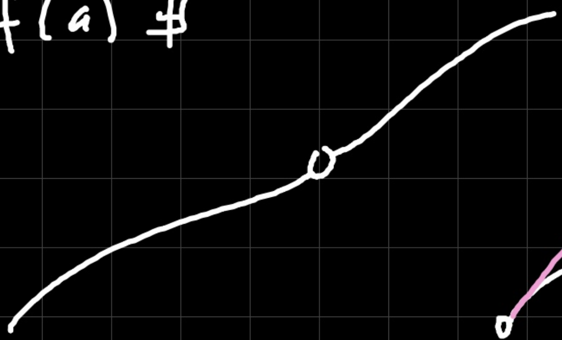
$$f'(x) = \begin{cases} 1 & x > 2 \\ -1 & x < 2 \end{cases}$$

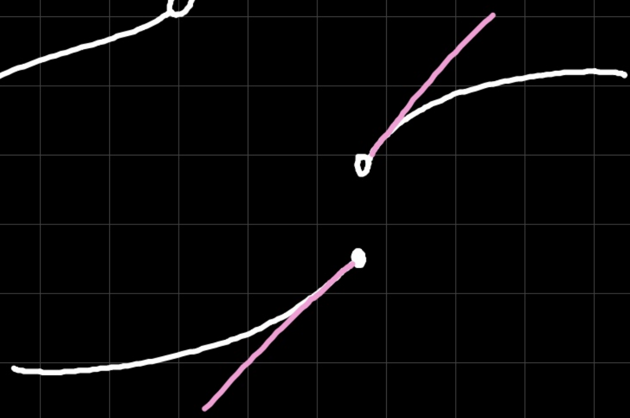
Since $f'_+(2) = 1$ but $f'_-(2) = -1$

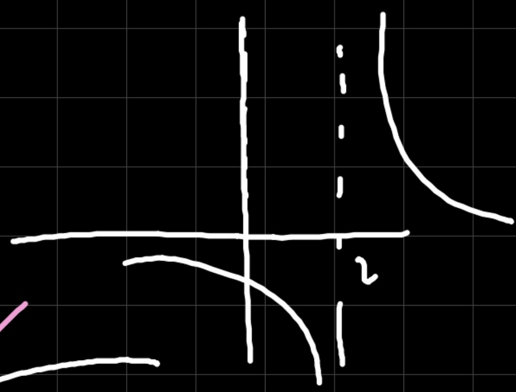
$f'(2) \nexists$ $\therefore$ f not diff. at $x = 2$.

Continuity Issue

$f(a) \nexists$







$\lim_{x \to a} f(x) \nexists$

$$f(x) = \begin{cases} x^3 - 3x & x \le 1 \\ 5x & x > 1 \end{cases}$$

$3x^2 - 3 \to 0$

$5 \to 5$

__1 way__

$\lim_{x \to 1^+} f(x) = 5$ but

$\lim_{x \to 1^-} f(x) = -2$

$\therefore \lim_{x \to 1} f(x) \nexists \therefore f$ discon

at $x = 1 \therefore f$ not diff at $x=1$

__2nd way__

$$f'(x) = \begin{cases} 3x^2 - 3 & x \ge 1 \\ 5 & x > 1 \end{cases}$$

Since $f'_+(1) = 5$ but

$f'_-(1) = 0$, $f'(1) \nexists$

$\therefore f$ not diff. at $x = 1$

$$f(x) = \begin{cases} x^2+3 & x \le 2 \\ x^2-4 & x > 2 \end{cases}$$

$2x \to 4$

$2x \to 4$

Cont test at $x=2$

$f(2) = 7$

$$\left.\begin{array}{l} \lim\limits_{x \to 2^+} f(x) = 0 \\ \lim\limits_{x \to 2^-} f(x) = 7 \end{array}\right\} \therefore \lim\limits_{x \to 2} f(x) \nexists$$

Since $\lim\limits_{x \to 2} f(x) \nexists$ f is discon at $x=2$ $\therefore$ f not diff at $x=2$.

$$f(x)=\begin{cases} x^2+1 & x\le 1 \\ 2x & x>1 \end{cases}$$

$2x \to 2$

$2 \to 2$

Cont test at $x=1$

$f(1)=2$

$$\left.\begin{array}{l}\lim_{x\to 1^+} f(x)=2 \\ \lim_{x\to 1^-} f(x)=2\end{array}\right\} \therefore \lim_{x\to 1} f(x)=2$$

Since $f(1)=\lim_{x\to 1} f(x)$ f is cont at $x=1$.

Deriv.

$$f'(x)=\begin{cases} 2x & x<1 \\ 2 & x>1 \end{cases}$$

Since $f'_+(1)=2$ and $f'_-(1)=2$ and f is cont at $x=1$, f is diff at $x=1$.

$$f(x) = \begin{cases} x^2+2 & x \leq 1 \\ x+2 & x > 1 \end{cases}$$

$2x \to 2$

$1 \to 1$

$$f'(x) = \begin{cases} 2x & x < 1 \\ 1 & x > 1 \end{cases}$$

Since $f'_+(1) = 1$ but

$f'_-(1) = 2$, $f'(1) \nexists$

$\therefore$ not diff at $x = 1$.

$$f(x) = \begin{cases} x^2 & x < 1 \\ ax + b & x \geq 1 \end{cases}$$

find a & b so that f is diff.

<u>Cont test at $x = 1$</u>

$f(1) = 1$

$\lim_{x \to 1^+} f(x) = a + b$

$\lim_{x \to 1^-} f(x) = 1$

for f to be cont at $x = 1$,

$a + b = 1$

<u>Deriv</u>

$$f'(x) = \begin{cases} 2x & x < 1 \\ a & x > 1 \end{cases}$$

$f'_-(1) = 2$ and

$f'_+(1) = a$

$\therefore a = 2$

$\therefore$ for f to be diff at $x = 1$,

$a = 2$ and $b = -1$

$$f(x) = \begin{cases} 5x-2 & x \leq -2 \\ x^2 & -2 < x < 3 \\ x^3 & x \geq 3 \end{cases}$$

$5 \to 5$

$2x \to 4 \quad 9$

$3x^2 \quad 27$

$$f'(x) = \begin{cases} 5 & x < -2 \\ 2x & -2 < x < 3 \\ 3x^2 & x > 3 \end{cases}$$

Since $f'_+(-2) = -4$ but $f'_-(-2) = 5 \to f'(-2) \nexists$

$\therefore$ f not diff at $x = -2$

Since $f'_+(3) = 27$ but $f'_-(3) = 6 \to f'(3) \nexists$

$\therefore$ f not diff at $x = 3$.