Differentiation Theorems

$D_x[c] = 0$

$D_x[x^n] = n x^{n-1}$ Power Rule $D_x[\sqrt{x}] = \frac{1}{2\sqrt{x}}$

$f(x) = x^7 \longrightarrow f'(x) = 7x^6$

$g(x) = 8x^3 \longrightarrow g'(x) = 24x^2$

$h(x) = \sqrt{x^5} = x^{5/2} \longrightarrow h'(x) = \frac{5}{2} x^{3/2} = \frac{5\sqrt{x^3}}{2}$

$f(x) = \sqrt{x} = x^{1/2} \longrightarrow f(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

Product Rule

$$D_x\left[f(x)\,g(x)\right] = f(x)\,g'(x) + g(x)\,f'(x)$$

$f(x) = (3x-5)(2x+1)$

$f'(x) = (3x-5)(2) + (2x+1)(3)$

$= 6x - 10 + 6x + 3$

$= 12x - 7$

$f(x) = 6x^2 - 7x - 5$

$f'(x) = 12x - 7$

$g(x) = \sqrt{x}\,(5x^2-4)$ $\quad g'(x) = (\sqrt{x})(10x) + (5x^2-4)\left(\frac{1}{2\sqrt{x}}\right)$

Quotient Rule

$$D_x\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$f(x) = \frac{3x-1}{2x+3}$$

$$f'(x) = \frac{(2x+3)(3) - (3x-1)(2)}{(2x+3)^2}$$

$$= \frac{6x+9-6x+2}{(2x+3)^2}$$

$$= \frac{11}{(2x+3)^2}$$

$$f(x) = \frac{3x-6}{\sqrt[3]{x}}$$

$$= x^{-1/3}(3x-6)$$

$$= 3x^{2/3} - 6x^{-1/3}$$

$$f'(x) = 2x^{-\frac{1}{3}} + 2x^{-4/3}$$

$$= \frac{2}{\sqrt[3]{x}} + \frac{2}{\sqrt[3]{x^4}}$$

$$f(x) = x^{23} - 30x^2 + 3$$

$$f'(x) = 23x^{22} - 60x$$

$$t(w) = \frac{\sqrt{5}}{w^5}$$

$$= \sqrt{5}\, w^{-5}$$

$$t'(w) = -5\sqrt{5}\, w^{-6}$$

$$= -\frac{5\sqrt{5}}{w^6}$$

$$u = v^4 - \sqrt[4]{v}$$

$$= v^4 - v^{1/4}$$

$$\frac{du}{dv} = 4v^3 - \frac{1}{4}v^{-3/4}$$

$$= 4v^3 - \frac{1}{4\sqrt[4]{v^3}}$$

WRITING EQ OF TAN

I. Write eq of normal to $y = \frac{10}{14-x^2}$ at $x=4$.

$y(4) = -5 \longrightarrow (4,-5)$

$$\frac{dy}{dx} = \frac{(14-x^2)(0)-(10)(-2x)}{(14-x^2)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=4} = \frac{80}{4} = 20 \longrightarrow m_T = 20 \longrightarrow m_\perp = -\frac{1}{20}$$

$-\frac{1}{20}$

$$\therefore y+5 = 20(x-4)$$

II. Write eq. of tan to $y = 3x^2 - 4x$ that is parallel to $2x - y + 3 = 0$.

Slope of given line is 2 $\longrightarrow m_T = 2$

$$\frac{dy}{dx} = 6x - 4$$

$$\therefore 6x - 4 = 2 \longrightarrow x = 1 \longrightarrow y = -1 \quad (1, -1)$$

$$\therefore y + 1 = 2(x - 1)$$

III Write eq tan to $y = x^2$ that passes thru $(5,9)$.

$$\frac{dy}{dx} = 2x$$

Slope of line thru (x, x^2) & $(5,9)$ is $\frac{x^2-9}{x-5}$

$$\therefore 2x = \frac{x^2-9}{x-5} \longrightarrow x = 1 \text{ or } x = 9$$

$$\therefore (1,1) \text{ and } (9,81)$$

At $(1,1)$ $\left.\frac{dy}{dx}\right|_{x=1} = 2 \longrightarrow y - 1 = 2(x-1)$

At $(9,81)$ $\left.\frac{dy}{dx}\right|_{x=9} = 18 \longrightarrow y - 81 = 18(x-9)$