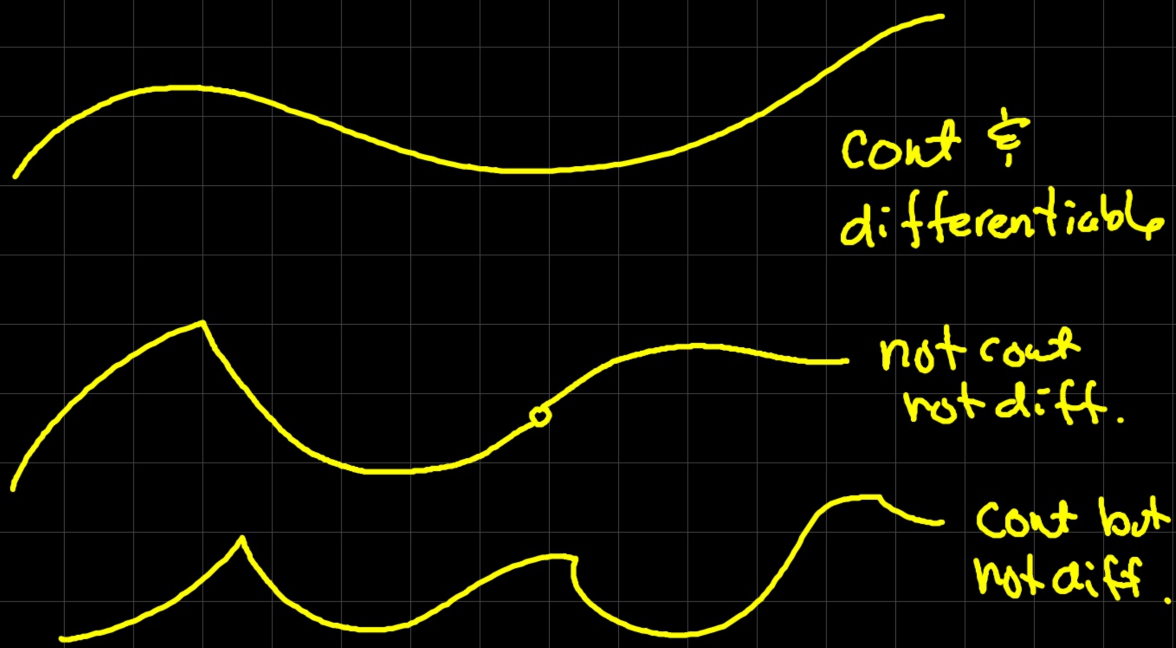
Differentiability



Is $f(x) = \sqrt{x}$ diff at $x=0$?

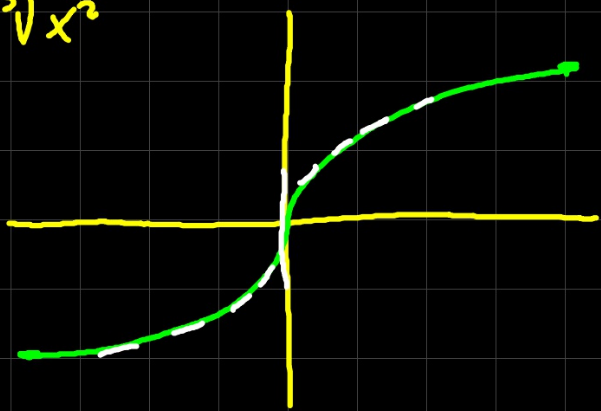
$$f'(x) = \frac{1}{2\sqrt{x}}$$

Since $f'(0) \nexists$ f is not diff at $x=0$.

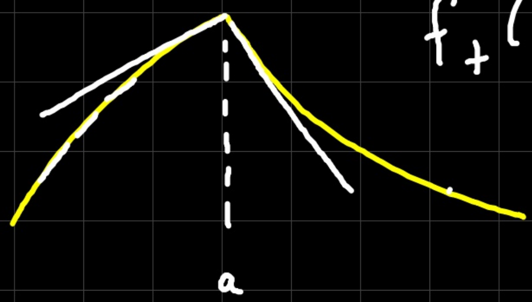
Vertial Tangat

$$f(x) = \sqrt[3]{x}$$

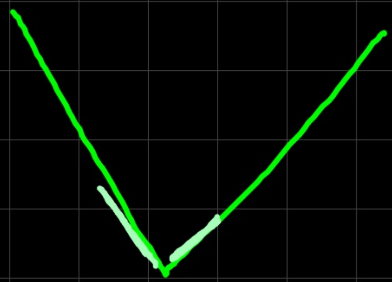
$$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$$



Corners / Cusps



$f'_+(a) \neq f'_-(a)$



$$f(x) = |x-2| = \begin{cases} x-2 & x \geq 2 \\ 2-x & x < 2 \end{cases}$$

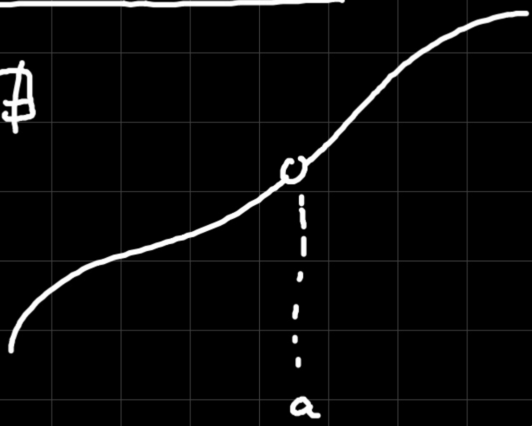
$$f'(x) = \begin{cases} 1 & x > 2 \\ -1 & x < 2 \end{cases}$$

f is not diff. at $x=2$ because

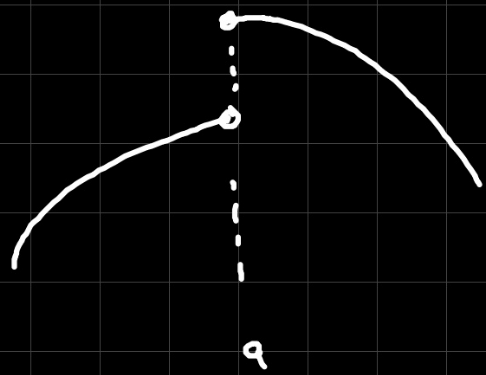
$f'_+(2) = 1$ but $f'_-(2) = -1$.

Continuity Issue

$f(a) \nexists$



$\lim_{x \to a} f(x) \nexists$



★ If f is diff → f is cont. ★

If f is cont → f is diff

not always true

$$f(x) = \begin{cases} x^3 - 3x & x \le 1 \\ 5x & x > 1 \end{cases}$$

$$f'(x) = \begin{cases} 3x^2 - 3 & x < 1 \\ 5 & x > 1 \end{cases}$$

f is not diff at $x = 1$ because

$f'_+(1) = 5$ but $f'_-(1) = 1$.

$2x \to 4$

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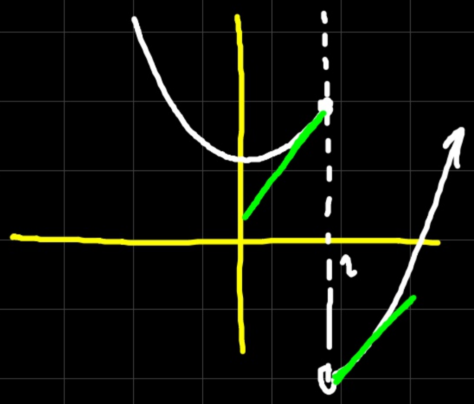
$$f(x) = \begin{cases} x^2+3 & x \le 2 \\ x^2-4 & x > 2 \end{cases}$$

$$f'(x) = \begin{cases} 2x & x < 2 \\ 2x & x > 2 \end{cases}$$



Cont test at $x=2$

$f(2) = 7$

$$\left.\begin{array}{l} \lim\limits_{x \to 2+} f(x) = 0 \\ \lim\limits_{x \to 2-} f(x) = 7 \end{array}\right\} \therefore \lim_{x \to 2} f(x) \nexists$$

$\therefore$ f is not cont at $x=2$ $\therefore$ f not diff at $x=2$.

$$f(x) = \begin{cases} x^2+1 & x \le 1 \\ 2x & x > 1 \end{cases}$$

$2x \to 2$ ✓
$2 \to 2$ ✓

<u>Cont test at $x=1$</u>

$f(1) = 2$

$$\left.\begin{array}{l} \lim_{x \to 1^+} f(x) = 2 \\ \lim_{x \to 1^-} f(x) = 2 \end{array}\right\} \therefore \lim_{x \to 1} f(x) = 2$$

f is cont at $x=1$ becau $f(1) = \lim_{x \to 1} f(x)$

$$f'(x) = \begin{cases} 2x & x<1 \\ 2 & x>1 \end{cases}$$

f is diff at $x=1$ becau $f'_+(1) = 2$ and $f'_-(1) = 2$ <u>AND</u> f is cont at $x=1$.

Given $f(x) = \begin{cases} x^2 & x < 1 \\ ax+b & x \geq 1 \end{cases}$ find value of a & b

so that f is diff at $x=1$.

Cont test at $x=1$

$f(1) = a+b$

$\lim_{x \to 1^+} f(x) = a+b$

$\lim_{x \to 1^-} f(x) = 1$

$\therefore$ for f to be cont at $x=1$,

$a+b=1$

Deriv

$f'(x) = \begin{cases} 2x & x < 1 \\ a & x > 1 \end{cases}$

for f to be diff at $x=1$

$f'_+(1) = f'_-(1) \rightarrow a=2$

$\therefore a=2 \rightarrow b=-1$

$$f(x) = \begin{cases} 5x-2 & x \le -2 \\ x^2 & -2 < x < 3 \\ x^3 & x \ge 3 \end{cases}$$

	-2	3
5	$\to 5$	
$2x$	$\to -4$	6
$3x^2$		27

$$f'(x) = \begin{cases} 5 & x < -2 \\ 2x & -2 < x < 3 \\ 3x^2 & x > 3 \end{cases}$$

Since $f'_+(-2) = -4$ but $f'_-(-2) = 5$,
f is not diff at $x = -2$.

Since $f'_+(3) = 27$ but $f'_-(3) = 6$,
f is not diff at $x = 3$.