The Chain Rule

$$h(x) = f(g(x))$$

$$h'(x) = \lim_{x_1 \to x} \frac{h(x_1) - h(x)}{x_1 - x}$$

$$= \lim_{x_1 \to x} \frac{f(g(x_1)) - f(g(x))}{x_1 - x} \cdot \frac{g(x_1) - g(x)}{g(x_1) - g(x)}$$

$$= \lim_{x_1 \to x} \frac{f(g(x_1)) - f(g(x))}{g(x_1) - g(x)} \cdot \frac{g(x_1) - g(x)}{x_1 - x}$$

$$= \boxed{\lim_{g(x_1) \to g(x)} \frac{f(g(x_1)) - f(g(x))}{g(x_1) - g(x)}} \cdot \boxed{\lim_{x_1 \to x} \frac{g(x_1) - g(x)}{x_1 - x}}$$

$$= f'(g(x)) \cdot g'(x).$$

CHAIN RULE

If $h(x) = f(g(x)) \longrightarrow h'(x) = f'(g(x))\,g'(x)$.

$$f(x) = (5x-3)^8$$

$$f'(x) = 8(5x-3)^7\,5 = 40(5x-3)^7$$

$$g(x) = \sin x^3$$

$$g'(x) = (\cos x^3)(3x^2)$$

$$= 3x^2 \cos x^3$$

$$h(x) = \tan 8x$$

$$h'(x) = 8 \sec^2 8x$$

$$f(x) = \sqrt{5x-4}$$

$$f'(x) = \frac{5}{2\sqrt{5x-4}}$$

$$g(x) = \sqrt{\sin 3x}$$

$$g'(x) = \frac{3\cos 3x}{2\sqrt{\sin 3x}}$$

$$D_x[c] = 0$$

$$D_x\left[f(x)^n\right] = n\, f(x)^{n-1} f'(x)$$ Power Rule

$$D_x\left[\sqrt{f(x)}\right] = \frac{f'(x)}{2\sqrt{f(x)}}$$

Product / Quotient Rules written same –

$$D_x[\sin f(x)] = f'(x) \cos f(x)$$

$$D_x[\cot f(x)] = -f'(x) \csc^2 f(x)$$

$$D_x[\cos f(x)] = -f'(x) \sin f(x)$$

$$D_x[\sec f(x)] = f'(x) \sec f(x) \tan f(x)$$

$$D_x[\tan f(x)] = f'(x) \sec^2 f(x)$$

$$D_x[\csc f(x)] = -f'(x) \csc f(x) \cot f(x)$$

Another Version of Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Ex $\quad y = u^3 \qquad u = 5x - 7 \qquad$ find $\frac{dy}{dx}$.

$$\frac{dy}{du} = 3u^2 \qquad \frac{du}{dx} = 5$$

$$\frac{dy}{dx} = 3u^2(5)$$

$$= 3(5x-7)^2(5)$$

$$y = (5x-7)^3$$

$$\frac{dy}{dx} = 3(5x-7)^2(5)$$

$f(x) = x^3 \sin x$

x^3

$3x^2 \cdot 1$

$f'(x) = x^3 \cos x + 3x^2 \sin x$

$f(x) = x^3 \sin^4 x$

$f'(x) = (x^3)(4\sin^3 x)(\cos x) + 3x^2 \sin^4 x$

$g(x) = \sec^8 x^3$

$g'(x) = [8\sec^7 x^3][\sec x^3 \tan x^3](3x^2)$

$$h(x) = \frac{11}{x^2+5x+6}$$

$$= 11\left(x^2+5x+6\right)^{-1}$$

$$h'(x) = -11\left(x^2+5x+6\right)^{-2}\left(2x+5\right)$$

$$= -\frac{11(2x+5)}{(x^2+5x+6)^2}$$

$$f(x) = \sqrt[4]{\sin x} = (\sin x)^{1/4}$$

$$f'(x) = \frac{1}{4}(\sin x)^{-3/4}(\cos x)$$

$$= \frac{\cos x}{4\sqrt[4]{\sin^3 x}}$$

$$\sqrt{x} \longrightarrow \frac{1}{2\sqrt{x}}$$

$$\sqrt[3]{x} \longrightarrow \frac{1}{3\sqrt[3]{x^2}}$$

$$f(x) = \sqrt{x^2+2x}$$

$$f'(x) = \frac{2x+2}{2\sqrt{x^2+2x}}$$

$$x^{1/3} \rightarrow \frac{1}{3}x^{-2/3}$$

$$\frac{1}{3\sqrt[3]{x^2}}$$

$$x^{\frac{1}{4}} \rightarrow \frac{1}{4}x^{-3/4}$$

$$\frac{1}{4\sqrt[4]{x^3}}$$

$$f(x) = \sqrt[5]{x^2-x}$$

$$f'(x) = \frac{2x-1}{5\sqrt[5]{(x^2-x)^4}}$$

$$D_x\left[\sqrt[n]{f(x)}\right] = \frac{f'(x)}{n\sqrt[n]{f(x)^{n-1}}}$$

$$f(x) = \sqrt{3 + \sqrt{x}} \qquad f(g(x)) \longrightarrow f'(g(x))\, g'(x)$$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}}{2\sqrt{3+\sqrt{x}}}$$

$$f(x) = g(x^3)$$

$$f'(x) = g'(x^3)\ 3x^2$$

$$k(x) = f(g(h(x)))$$

$$k'(x) = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$$