The Derivative of the Natural Exponential Function

$$\boxed{D_x\left[e^{f(x)}\right]}$$

$$D_x\left[x^n\right] = n x^{n-1}$$

$$D_x\left[a^{f(x)}\right]$$

$$D_x\left[f(x)^n\right] = n f(x)^{n-1} f'(x)$$

$$D_x\left[\sin x\right] = \cos x$$

$$D_x\left[\ln f(x)\right]$$

$$D_x\left[\sin f(x)\right] = \cos f(x)\, f'(x)$$

$$D_x\left[\log_a f(x)\right]$$

$$f(x) = e^x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x e^h - e^x}{h}$$

$$= \lim_{h \to 0} e^x \frac{e^h - 1}{h}$$

$$= e^x \lim_{h \to 0} \frac{e^h - 1}{h}$$

$$= e^x$$

$$D_x\left[e^x\right]=e^x$$

$$D_x\left[e^{f(x)}\right]=e^{f(x)}f'(x)=f'(x)e^{f(x)}$$

$$f(x)=e^{x^3}\longrightarrow f'(x)=3x^2e^{x^3}$$

$$g(x)=e^{\sin x}\longrightarrow g'(x)=e^{\sin x}\cos x$$

$$h(x)=e^{\sin x^3}\longrightarrow h'(x)=3x^2e^{\sin x^3}\cos x^3$$

$$f(x) = x^2 e^{x^2}$$

$$f'(x) = (x^2)(2x e^{x^2}) + (e^{x^2})(2x)$$

$$= 2x e^{x^2}(x^2 + 1)$$

$$e^{xy} - x^3 = 1 - 3y^2 \quad \text{find } \frac{dy}{dx}.$$

$$e^{xy}\left(x\frac{dy}{dx} + y\right) - 3x^2 = -6y\frac{dy}{dx}$$

$$xe^{xy}\frac{dy}{dx} + ye^{xy} - 3x^2 = -6y\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3x^2 - ye^{xy}}{xe^{xy} + 6y}$$

Write eq tan to $2e^{xy} = x+y$ at $(0,2)$.

$$2e^{xy}\left(x\frac{dy}{dx}+y\right) = 1+\frac{dy}{dx}$$

$$2xe^{xy}\frac{dy}{dx}+2ye^{xy} = 1+\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1-2ye^{xy}}{2xe^{xy}-1} \rightarrow \left.\frac{dy}{dx}\right|_{(0,2)} = \frac{-3}{-1} = 3$$

$$\therefore y-2 = 3(x-0)$$

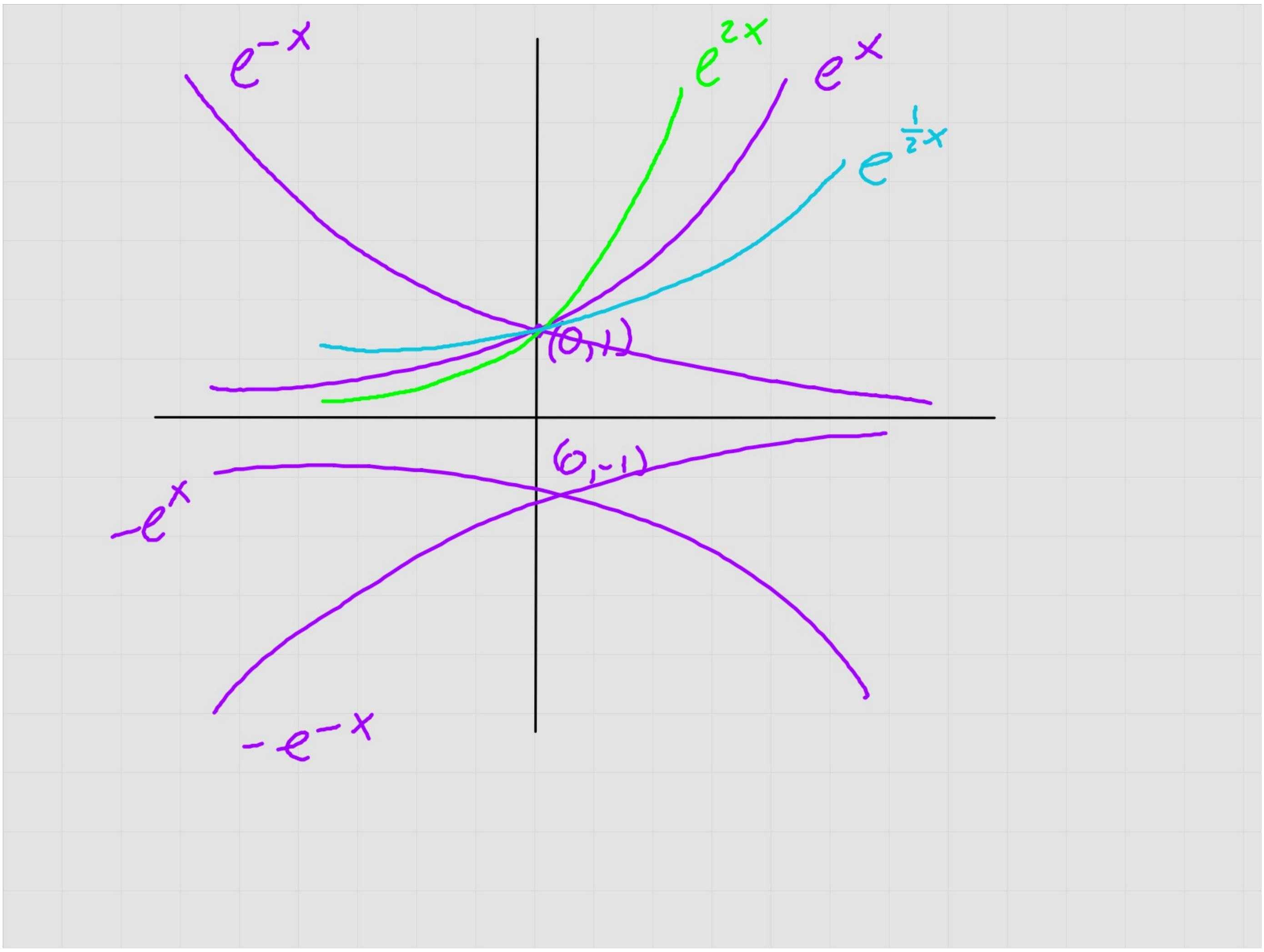
$$f(x) = \sqrt{e^{3x} + 2x}$$

$$f'(x) = \frac{3e^{3x} + 2}{2\sqrt{e^{3x} + 2x}}$$

$$\lim_{x \to \infty} \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} = \lim_{x \to \infty} \frac{e^{2x} - \frac{1}{e^{2x}}}{e^{2x} + \frac{1}{e^{2x}}} \frac{e^{2x}}{e^{2x}} = 1$$

$$= \lim_{x \to \infty} \frac{e^{4x} - 1}{e^{4x} + 1}$$

$$= 1$$



e^{-x}
e^{2x}
e^{x}
$e^{\frac{1}{2}x}$
(0,1)
(0,-1)
$-e^{x}$
$-e^{-x}$