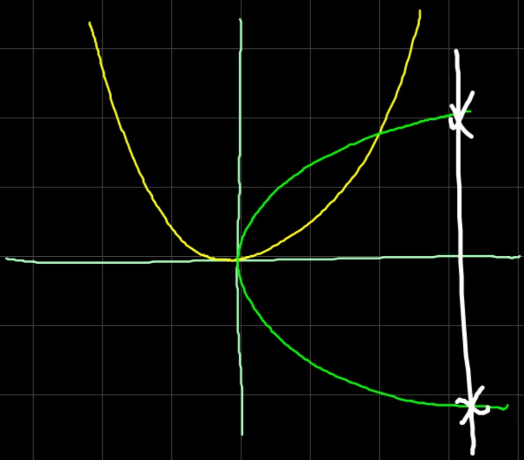
Inverse Functions



$(2,4)$ $(4,2)$

$y = x^2$ or $f(x) = x^2$

f $(2,4)$ f^{-1} $(4,2)$

f and g are inverse iff

$$f(g(x)) = x \quad \forall x$$

and

$$g(f(x)) = x \quad \forall x$$

$$f(f^{-1}(x)) = x \quad \forall x$$

and

$$f^{-1}(f(x)) = x \quad \forall x$$

$$x = f^{-1}(y)$$

$$y = f^{-1}(x)$$

Show that $f(x) = 3x-2$ and $g(x) = \frac{x+2}{3}$ are inverses.

$$f(g(x)) = f\left(\frac{x+2}{3}\right)$$
$$= 3\left(\frac{x+2}{3}\right) - 2$$
$$= x$$

$$g(f(x)) = g(3x-2)$$
$$= \frac{(3x-2)+2}{3}$$
$$= x$$

Since $f(g(x)) = x \ \forall x$

and $g(f(x)) = x \ \forall x$,

f and g are inv.

Find the inverse of $f(x) = \frac{x-2}{x+3}$.

- Let $y = f(x)$

$$y = \frac{x-2}{x+3}$$

$$xy + 3y = x - 2$$

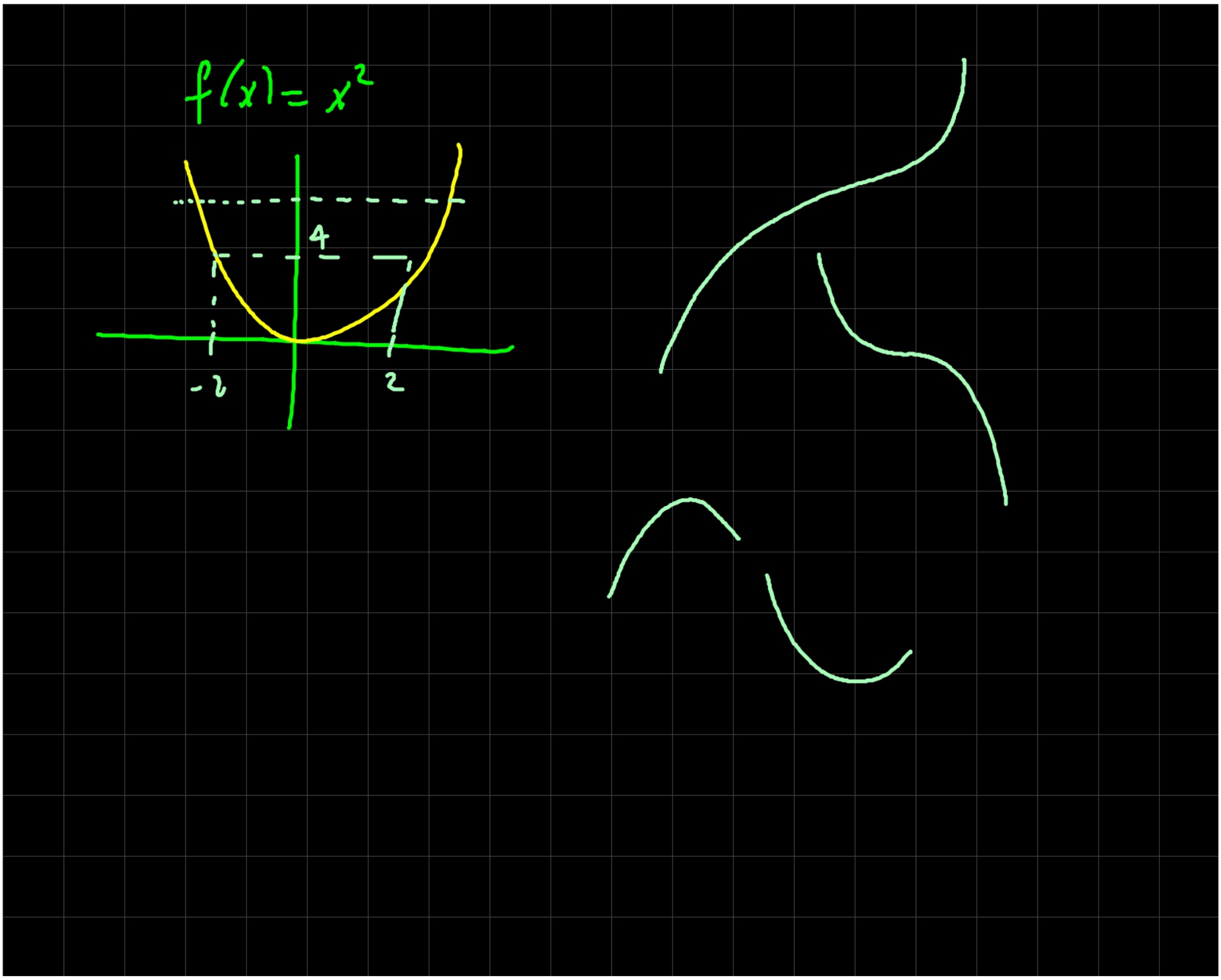
$$xy - x = -2 - 3y$$

$$x(y-1) = -2 - 3y$$

- $x = \frac{-2-3y}{y-1}$

- $f^{-1}(y) = \frac{-2-3y}{y-1}$

- $f^{-1}(x) = \frac{-2-3x}{x-1}$


$f(x) = x^2$
4
-2
2

f has an inv. iff f is monotonic.

f has an inv. iff $f'(x) \geq 0 \ \forall x$ in f

or if $f'(x) \leq 0 \ \forall x$ in f.

Determine if $f(x) = \frac{x-3}{x+2}$ has an inv.

$$f'(x) = \frac{5}{(x+2)^2}$$

Since $f'(x) \geq 0 \ \forall x$ in f, f has an inv.

Determine if $\boxed{f(x) = x^5 + 5x^3 + 2x - 4}$ has an inverse.

$$f'(x) = 5x^4 + 15x^2 + 2$$

Since $f'(x) \geq 0 \ \forall x$ in f, f has an inverse.

Can you find it?

let $y = f(x)$

$$y = x^5 + 5x^3 + 2x - 4$$

$(1,4)$ on f $\quad (4,1)$ f^{-1}

$(f^{-1})'(4)$ $\quad (f^{-1})'(x)$

$f^{-1}(x)$

$y - y_1 = m(x - x_1)$ $\quad$ f @ (c,d)

$y - d = f'(c)(x - c)$

$\frac{y - d}{f'(c)} = x - c$

$x - c = \frac{1}{f'(c)}(y - d)$

$y - c = \frac{1}{f'(c)}(x - d)$

$$\boxed{\text{If } (c,d) \text{ is on } f \text{ then } (f^{-1})'(d) = \frac{1}{f'(c)}}$$

Write eq of tan to f^{-1} at $x=4$ if

$$f(x) = x^5 + 5x^3 + 2x - 4.$$

$$x^5 + 5x^3 + 2x - 4 = 4 \longrightarrow x = 1$$

Since $(1,4)$ is on f then $(f^{-1})'(4) = \frac{1}{f'(1)}$

$$f'(x) = 5x^4 + 15x^2 + 2 \rightarrow f'(1) = 22 \longrightarrow (f^{-1})'(4) = \frac{1}{22}$$

We know $(4,1)$ on f^{-1}.

$$\boxed{y - 1 = \frac{1}{22}(x-4)}$$