Linearizations

(Tangent Line Approximations)

f @ $x = a$

$$y - y_1 = m(x - x_1)$$

$$y - f(a) = f'(a)(x - a)$$

$$y = f(a) + f'(a)(x - a)$$

$$\boxed{L(x) = f(a) + f'(a)(x - a)}$$

"linearization of f at $x = a$"

Write eq. of tan to
$f(x) = x^2 + 4x$ at $x = 1$
and use it to est $f(1.1)$.

$f(1) = 5$

$f'(x) = 2x + 4$

$f'(1) = 6$

$y - 5 = 6(x-1)$

$y = 5 + 6(x-1)$

$f(1.1) \approx y(1.1) = 5.600$

Linearize $f(x) = x^2 + 4x$ at $x = 1$
and use it to est $f(1.1)$

$f(1) = 5$

$f'(x) = 2x + 4 \rightarrow f'(1) = 6$

$L(x) = 5 + 6(x-1)$

$f(1.1) \approx L(1.1) = 5.600$

Write a linearization of $f(x)=\sqrt{x}$ that could be used to est $\sqrt{100.3}$.

Linearize f at $x=100$

$f(100)=10$

$f'(x)=\frac{1}{2\sqrt{x}} \longrightarrow f'(100)=\frac{1}{20}$

$L(x)=10+\frac{1}{20}(x-100)$

Use an appropriate linearization to est $\sqrt[3]{64.2}$.

Linearize $f(x) = \sqrt[3]{x}$ at $x = 64$

$$f(64) = 4$$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}} \longrightarrow f'(64) = \frac{1}{48}$$

$$L(x) = 4 + \frac{1}{48}(x - 64)$$

$$\sqrt[3]{64.2} \approx L(64.2) = 4 + \frac{1}{48}\left(\frac{2}{10}\right) = 4 + \frac{1}{240}$$