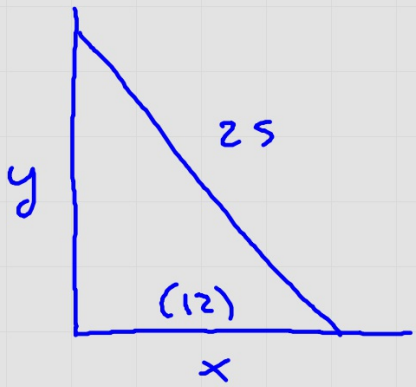
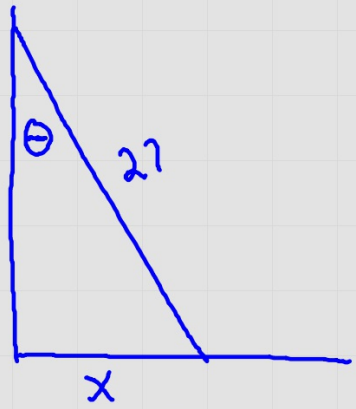
Related Rates

- write down any rates
- draw a diagram / label
- write eq'ties known & unknown.
  & know vol/area formulas
  · Py. Thm
  · similar Δ's
- diff. with respect to t
- plug n' chug

ladder

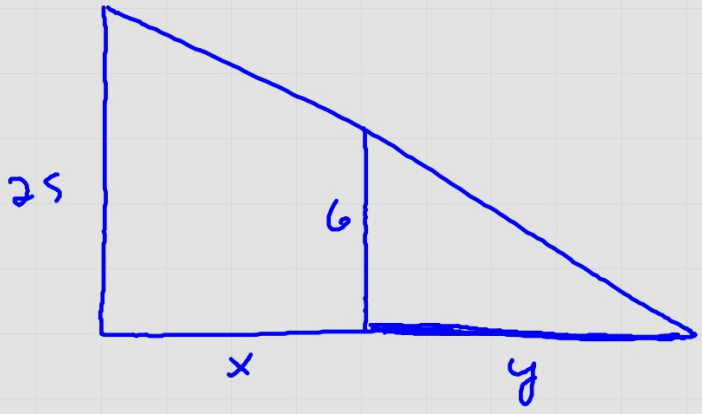




$$\sin\theta = \frac{1}{27}x$$

$$\cos\theta \frac{d\theta}{dt} = \frac{1}{27}\frac{dx}{dt}$$

Shadow



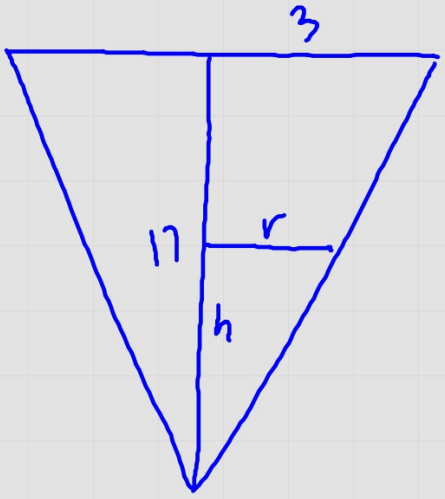
$$\frac{dx}{dt} = -2$$

$$\frac{25}{x+y} = \frac{6}{y}$$

$$6x + 6y = 25y$$

$$6x = 19y$$

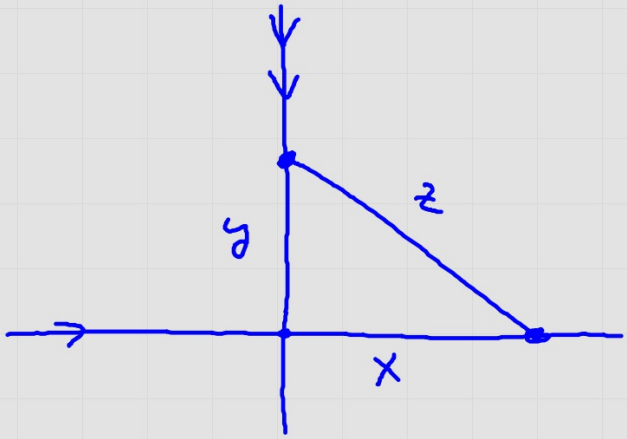
Cone



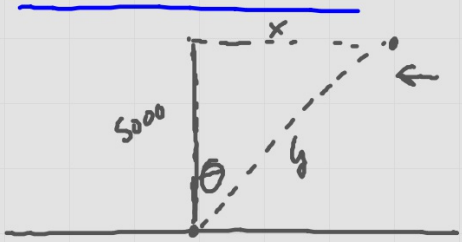
$V = \frac{1}{3}\pi r^2 h$

"dump the r"

Intersect



Kite / Airplane



① $\frac{dV}{dt} = 2$

Find $\frac{dr}{dt}$ when $r = .5$

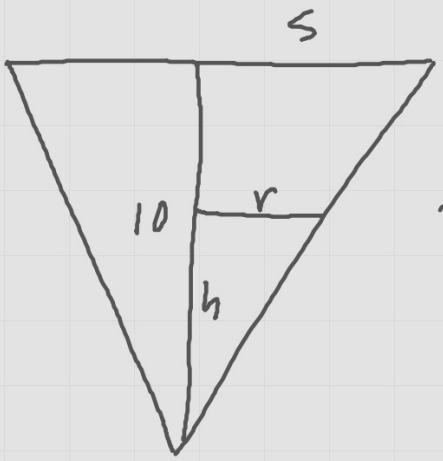
$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$2 = 4\pi(.5)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = .637$$

∴ the diameter incr. at 1.273 in/sec.

② 

$\frac{dV}{dt} = 3.14$

Find $\frac{dh}{dt}$ when $h = 7.5$

$\frac{10}{h} = \frac{5}{r}$

$10r = 5h$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \frac{h^2}{4} h$

$V = \frac{\pi}{12} h^3$

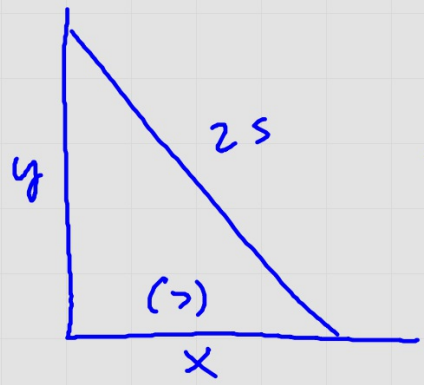
$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$3.14 = \frac{\pi}{4}(7.5)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = .071$

∴ water level rising at .071 ft/min.

③



$\frac{dz}{dt} = -1$

Find $\frac{dx}{dt}$ when $x = 7$

$y = \sqrt{625 - 49}$

$= 24$

$x^2 + y^2 = 25^2$

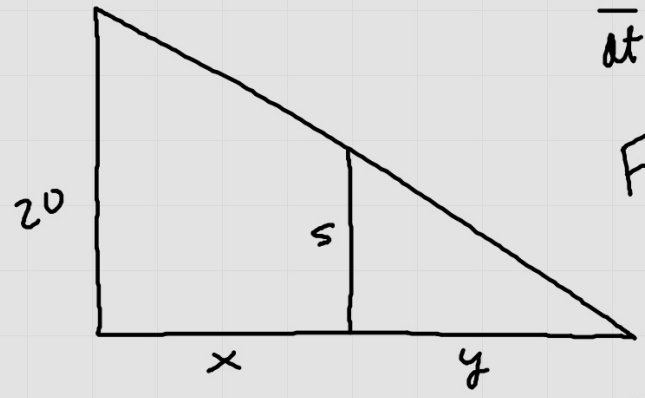
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$7\frac{dx}{dt} + (24)(-1) = 0$

$\frac{dx}{dt} = \frac{24}{7}$

∴ bottom moving away from the wall at $\frac{24}{7}$ ft/sec.

④



$\frac{dx}{dt} = -6$

Find $\frac{dy}{dt}$

$\frac{20}{x+y} = \frac{s}{y}$

$5x + 5y = 20y$

$5x = 15y$

$x = 3y$

$x = 3y$

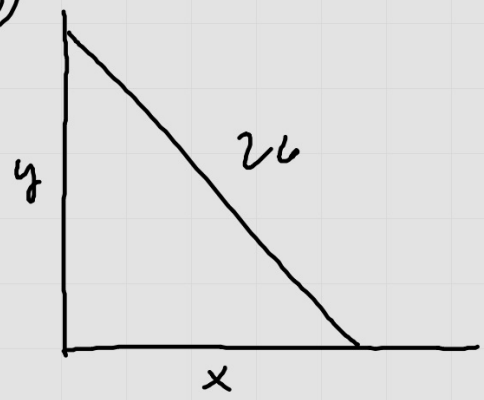
$\frac{dx}{dt} = 3\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{-6}{3} = -2$

∴ shadow getting shorter at 2 ft/sec.

∴ tip moving toward light at 8 ft/sec.

⑤



$y = \sqrt{26^2 - 24^2}$

$= 10$

$\frac{dx}{dt} = 2$

Find $\frac{dy}{dt}$ when $x = 24$

$x^2 + y^2 = 26^2$

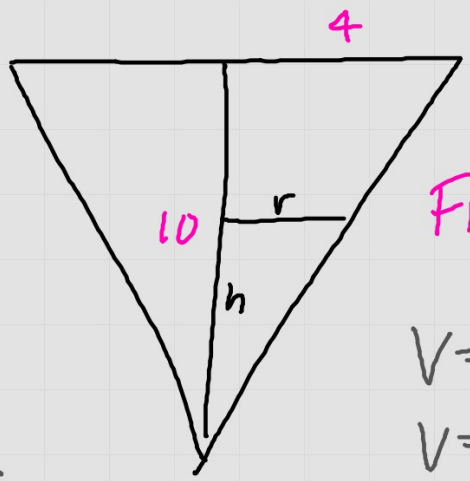
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$(24)(2) + (10)\frac{dy}{dt}$

$\frac{dy}{dt} = -4.800$

∴ top down wall at 4.800 ft/sec.

(6)



$\frac{dV}{dt} = 9$

Find $\frac{dh}{dt}$ when $h = 5$

$\frac{10}{h} = \frac{4}{r}$

$10r = 4h$

$r = \frac{2h}{5}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \frac{4h^2}{25} h$

$V = \frac{4\pi}{75} h^3$

$\frac{dV}{dt} = \frac{4\pi}{25} h^2 \frac{dh}{dt}$

$9 = \frac{4\pi}{25}(25)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{9}{4\pi}$

$= .716$

$\therefore$ ht of H_2O inc. at .716 ft/min