Implicit Differentiation

Explicitly defined

$y = x^2 + 5x$

$y = x^2 \sin x$

$f(x) = \sqrt{x-2}$

Implicitly defined

$x^3 + y^3 = 6xy$

$\cos(x+y) = x$

$y \quad \frac{dy}{dx}$

$D_x[x^3] = 3x^2$

$D_x[f(x)^3] = 3f(x)^2 f'(x)$

$D_x[y^3] = 3y^2 \frac{dy}{dx}$

$$x^2y^3 = x + 4y \quad \text{find } \frac{dy}{dx}.$$

$$3x^2y^2\frac{dy}{dx} + 2xy^3 = 1 + 4\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1 - 2xy^3}{3x^2y - 4}$$

$$\sin(x+y) = y^2 \cos x$$

$$\cos(x+y)\left[1 + \frac{dy}{dx}\right] = -y^2 \sin x + 2y \cos x \frac{dy}{dx}$$

$$\cos(x+y) + \cos(x+y)\frac{dy}{dx} = -y^2 \sin x + 2y \cos x \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-y^2 \sin x - \cos(x+y)}{\cos(x+y) - 2y \cos x}$$

$$\frac{dy}{dx} = \frac{y^2 \sin x + \cos(x+y)}{2y \cos x - \cos(x+y)}$$

$$D_x[x^4] = 4x^3$$

$$D_y[x^4] = 4x^3 \frac{dx}{dy}$$

$$D_t[x^4] = 4x^3 \frac{dx}{dt}$$

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

Write an eq of tan to $x^3+y^3=6xy$ at $(3,3)$.

$$3x^2 + 3y^2\frac{dy}{dx} = 6x\frac{dy}{dx} + 6y$$

$$\frac{dy}{dx} = \frac{6y-3x^2}{3y^2-6x}$$

$$\left.\frac{dy}{dx}\right|_{(3,3)} = \frac{18-27}{27-18} = -1 \longrightarrow m_T = -1$$

$$\therefore y-3 = -(x-3)$$