The Derivative of the Natural Exponential Function

$D_x\left[e^{f(x)}\right]$

$D_x[\sin x] = \cos x$

$D_x\left[a^{f(x)}\right]$

$D_x[\sin f(x)] = f'(x)\cos f(x)$

$D_x[\ln f(x)]$

$D_x[x^n] = n x^{n-1}$

$D_x[f(x)^n] = n f(x)^{n-1} f'(x)$

$D_x[\log_a f(x)]$

$$f(x) = e^x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x e^h - e^x}{h}$$

$$= \lim_{h \to 0} e^x \frac{e^h - 1}{h}$$

$$= e^x \cancel{\boxed{\lim_{h \to 0} \frac{e^h - 1}{h}}}\ 1$$

$$= e^x$$

$$D_x\left[e^{f(x)}\right] = e^{f(x)} f'(x) = f'(x)e^{f(x)}$$

$$f(x) = e^{x^3} \qquad f'(x) = 3x^2 e^{x^3}$$

$$g(x) = e^{\sin x} \qquad g'(x) = e^{\sin x}\cos x$$

$$h(x) = e^{\sin x^3} \qquad h'(x) = 3x^2 e^{\sin x^3} \cos x^3$$

$$f(x) = x^2 e^{x^2}$$

$$f'(x) = (x^2)(2x e^{x^2}) + (e^{x^2})(2x)$$

$$= 2x e^{x^2}(x^2 + 1)$$

$$e^{xy} - x^3 = 1 - 3y^2 \quad \text{find } \frac{dy}{dx}.$$

$$e^{xy}\left(x\frac{dy}{dx} + y\right) - 3x^2 = -6y\frac{dy}{dx}$$

$$xe^{xy}\frac{dy}{dx} + ye^{xy} - 3x^2 = -6y\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3x^2 - ye^{xy}}{xe^{xy} + 6y}$$

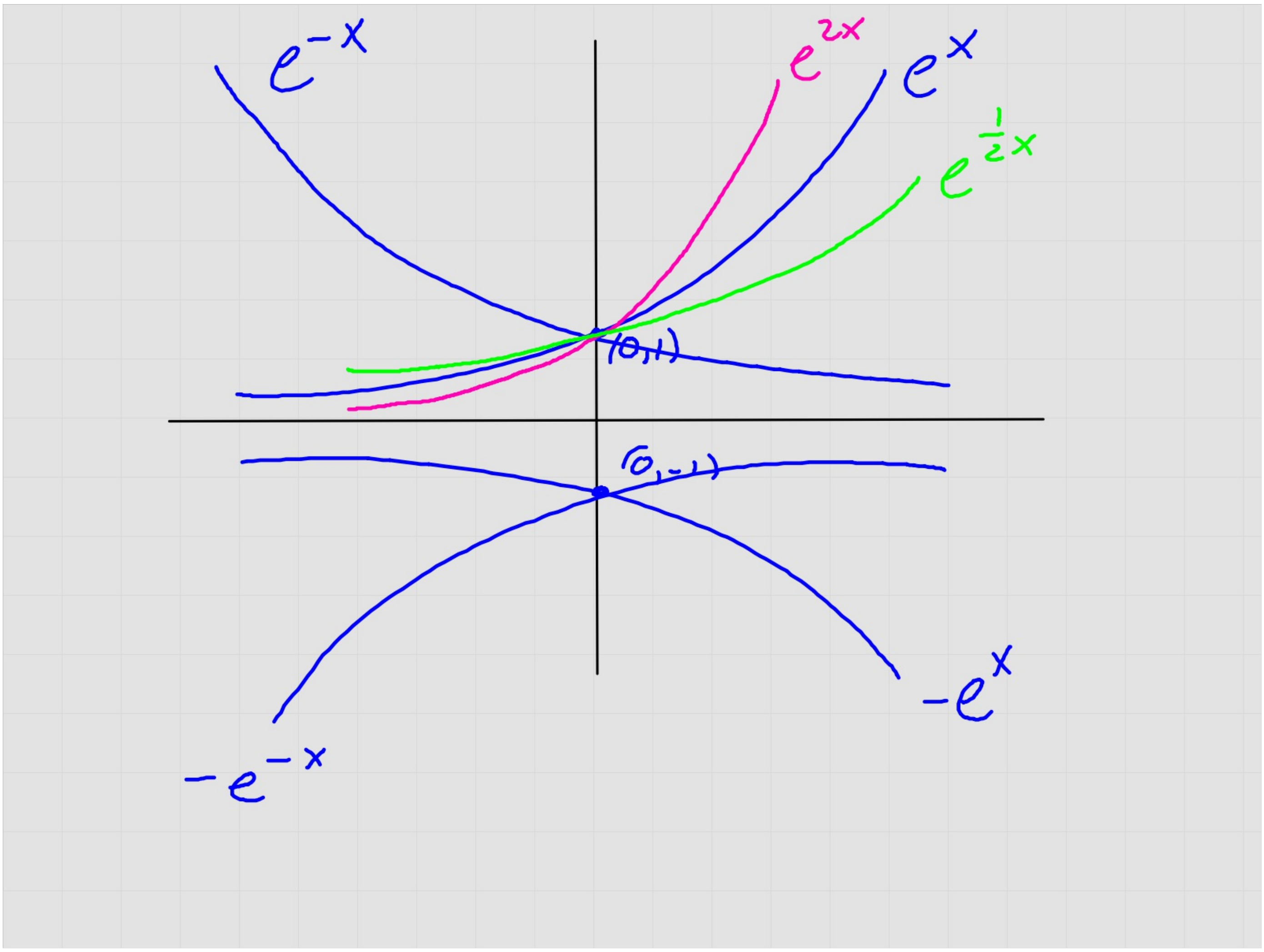
Write eq. tan to $e^{xy} = x + y$ at $(0, 2)$.

$$e^{xy}\left(x\frac{dy}{dx} + y\right) = 1 + \frac{dy}{dx}$$

$$xe^{xy}\frac{dy}{dx} + ye^{xy} = 1 + \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1 - ye^{xy}}{xe^{xy} - 1} \longrightarrow \left.\frac{dy}{dx}\right|_{(0,2)} = \frac{-1}{-1} = 1$$

$$\therefore\ y - 2 = 1(x - 0)$$



e^{-x}
e^{2x}
e^{x}
e^{\frac{1}{2}x}
(0,1)
(0,-1)
-e^{x}
-e^{-x}

$$\lim_{x\to\infty}\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}=\lim_{x\to\infty}\frac{e^{2x}-\frac{1}{e^{2x}}}{e^{2x}+\frac{1}{e^{2x}}}=1$$

$$\lim_{x\to-\infty}\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}=\lim_{x\to-\infty}\frac{e^{2x}-\frac{1}{e^{2x}}}{e^{2x}+\frac{1}{e^{2x}}}=-1$$