Linearizations

(Tangent Line Approximations)

f @ $x=a$

$$y - y_1 = m(x - x_1)$$

$$y - f(a) = f'(a)(x-a)$$

$$y = f(a) + f'(a)(x-a)$$

$$\boxed{L(x) = f(a) + f'(a)(x-a)}$$

"linearization of f at $x=a$"

Write eq tan to $f(x) = x^2 + 4x$ at $x = 1$ use it to est $f(1.1)$.

$f(1) = 5$

$f'(x) = 2x + 4$

$f'(1) = 6$

$y - 5 = 6(x-1)$

$y = 5 + 6(x-1)$

$f(1.1) \approx y(1.1) = 5 + 6\left(\frac{1}{10}\right)$

$= 5.600$

Linearize $f(x) = x^2 + 4x$ at $x = 1$ & use it to est $f(1.1)$.

$f(1) = 5$

$f'(x) = 2x + 4$

$f'(1) = 6$

$L(x) = 5 + 6(x-1)$

$f(1.1) \approx L(1.1) = 5.600$

Linearize $f(x) = \sin x$ at $x = 0$.

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$f(0) = 0$$

$$f'(x) = \cos x \rightarrow f'(0) = 1$$

$$L(x) = 0 + 1(x - 0)$$

$$L(x) = x$$

Write a linearization of $f(x) = \sqrt{x}$ that would be used to est. $\sqrt{81.3}$. $f(81.3)$

Linearize f at $x = 81$

$f(81) = 9$

$f'(x) = \frac{1}{2\sqrt{x}} \to f'(81) = \frac{1}{18}$

$L(x) = 9 + \frac{1}{18}(x - 81)$

Use an approp. linearization to approx $\sqrt[3]{64.1}$

Linearize $f(x) = \sqrt[3]{x}$ at $x = 64$.

$$f(64) = 4$$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}} \longrightarrow f'(64) = \frac{1}{48}$$

$$L(x) = 4 + \frac{1}{48}(x-64)$$

$$\sqrt[3]{64.1} \approx L(64.1) = 4 + \frac{1}{480} = 4.002$$