

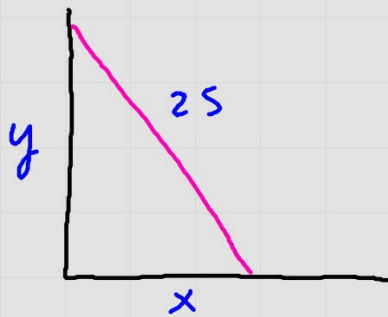
## Related Rates

$$V = \frac{4}{3} \pi r^3$$

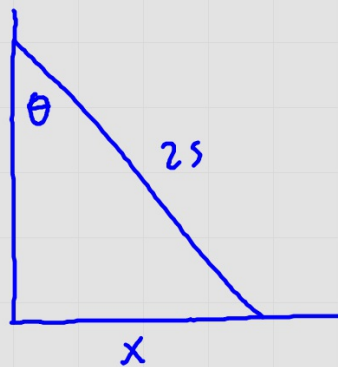
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

- 
- pull info from problem "...at a rate of..."  
& draw & label diag.  $\downarrow$   $\downarrow$
  - find eq that ties all knowns  $\frac{\quad}{dt}$   
& unknowns together  
Py.Thm, known area/vol, similar  $\Delta$ 's.

Ladder

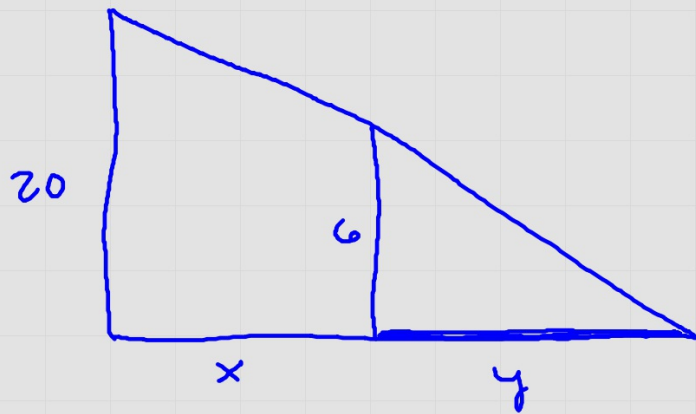


$$x^2 + y^2 = 2s$$



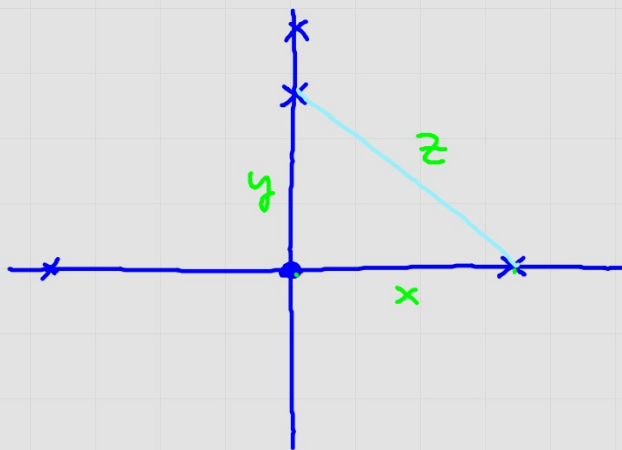
$$\sin \theta = \frac{x}{2s}$$

Shadow

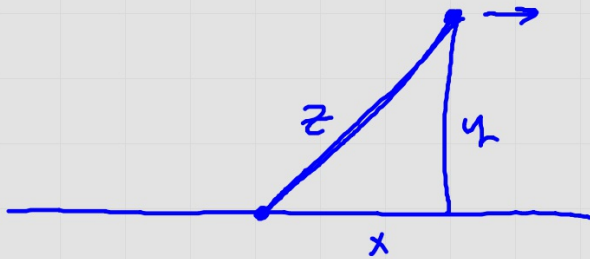


$$\frac{20}{x+y} = \frac{6}{y}$$

## Intersection



Kite / Airplane

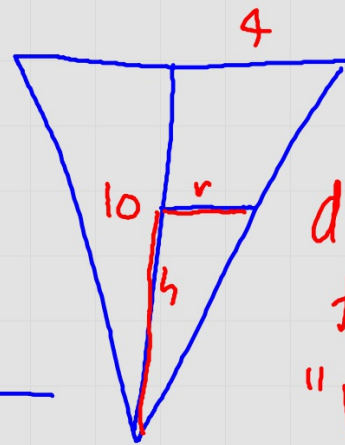
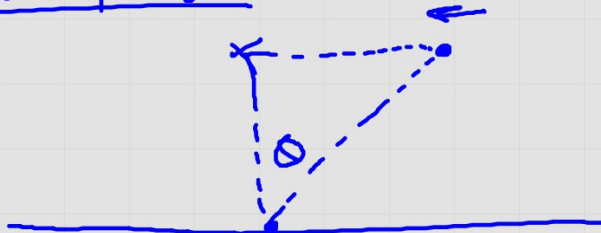


Circle

Sphere

Cone

Search / Light



dump  
the  
"r"

①

$$\frac{dV}{dt} = 2$$

Find  $\frac{dr}{dt}$  when  $r = .5$

$$V = \frac{4}{3}\pi r^3$$

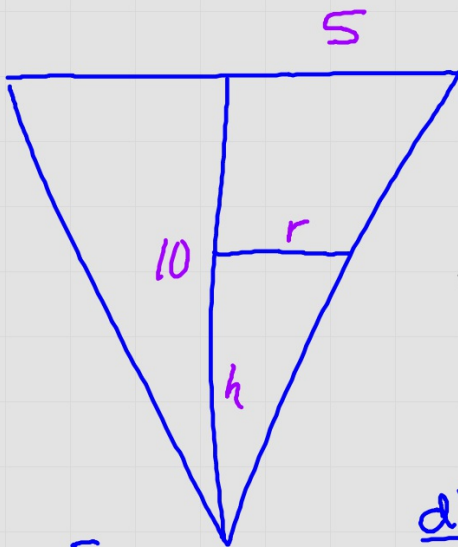
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$2 = 4\pi (.5)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = .637$$

$\therefore$  diameter is  
increasing at  
1.273 in/sec.

②



$$\frac{10}{h} = \frac{5}{r}$$

$$10r = 5h$$

$$r = \frac{h}{2}$$

$$\frac{dV}{dt} = 3.14$$

Find  $\frac{dh}{dt}$  when  $h = 7.5$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{\pi}{12} h^3$$

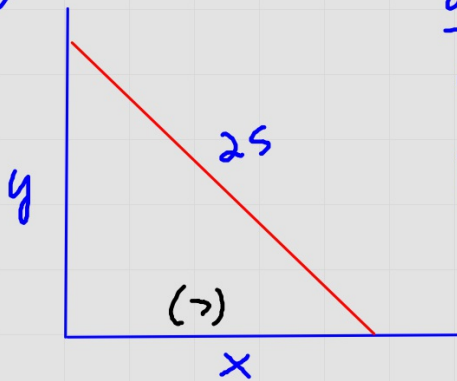
$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$3.14 = \frac{\pi}{4} (7.5)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = .071$$

$\therefore$  ht of  $H_2O$   
is increasing  
at .071 ft/min

③



$$y = \sqrt{625 - 49} \\ = 24$$

$$\frac{dy}{dt} = -1$$

Find  $\frac{dx}{dt}$  when  $x=7$

$$x^2 + y^2 = 25^2$$

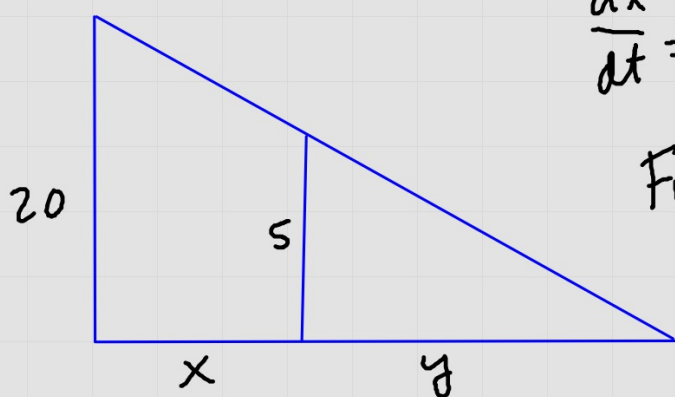
$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$7 \frac{dx}{dt} + (24)(-1) = 0$$

$$\frac{dx}{dt} = \frac{24}{7}$$

$\therefore$  bottom moving away at  $\frac{24}{7}$  ft/sec.

(4)



$$\frac{dx}{dt} = -6$$

Find  $\frac{dy}{dt}$

$$\begin{aligned}\frac{20}{x+y} &= \frac{5}{y} \\ 5x + 5y &= 20y \\ 5x &= 15y \\ x &= 3y\end{aligned}$$

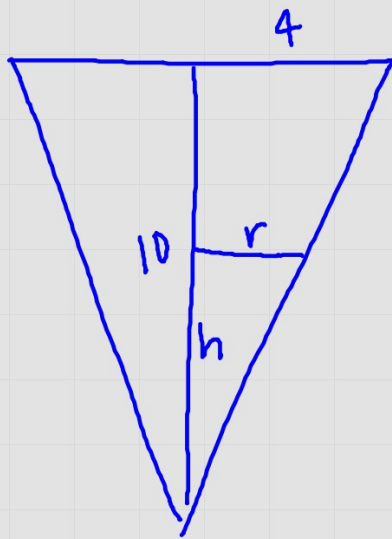
$$x = 3y$$

$$\frac{dx}{dt} = 3 \frac{dy}{dt}$$

$$\frac{dy}{dt} = -\frac{6}{3} = -2$$

$\therefore$  shadow getting shorter at 2 ft/sec.  
 $\therefore$  tip moving toward wall at 8 ft/sec.

(6)



$$\frac{10}{h} = \frac{4}{r}$$

$$10r = 4h$$

$$r = \frac{2h}{5}$$

$$\frac{dV}{dt} = 9$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \frac{4h^2}{25} h$$

$$V = \frac{4\pi}{75} h^3$$

$$\frac{dV}{dt} = \frac{4\pi}{25} h^2 \frac{dh}{dt}$$

$$9 = \frac{4\pi}{25} (5)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = .716$$

$\therefore$  H<sub>2</sub>O level  
rising at  
.716 ft/min