Implicit Differentiation

Explicitly defined

$y = x^2 + 2x$

$y = x \sin x$

$f(x) = \sqrt{x^2 - 2}$

Implicitly defined

$x^2 + y^3 = x - 3$

$x^3 + y^3 = 6xy$

$$D_x\left[f(x)^3\right] = 3f(x)^2 f'(x)$$

$$D_x\left[y^3\right] = 3y^2 \frac{dy}{dx}$$

$x \cdot y$

$x\frac{dy}{dx} + y$

$x^2 + y^3 = xy$ find $\frac{dy}{dx}$.

$$2x + 3y^2\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\frac{dy}{dx} = \frac{y - 2x}{3y^2 - x}$$

$$D_x\left[x^3\right] = 3x^2 \qquad D_w\left[a^7\right] = 7a^6\,\frac{da}{dw}$$

$$D_y\left[x^3\right] = 3x^2\,\frac{dx}{dy}$$

$x^3 + y^3 = 15$ diff. w/ resp. to t.

$$3x^2\,\frac{dx}{dt} + 3y^2\,\frac{dy}{dt} = 0$$

$$y^5 - 3x^2y^2 + 5x^4 = 12 \quad \text{find } \frac{dy}{dx}.$$

$$5y^4\frac{dy}{dx} - 3\left[2x^2y\frac{dy}{dx} + 2xy^2\right] + 20x^3 = 0$$

$$5y^4\frac{dy}{dx} - 6x^2y\frac{dy}{dx} - 6xy^2 + 20x^3 = 0$$

$$\frac{dy}{dx} = \frac{6xy^2 - 20x^3}{5y^4 - 6x^2y}$$

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\sin(x+y) = y^2 \cos x$$

$$\Rightarrow \cos(x+y)\left[1 + \frac{dy}{dx}\right] = -y^2 \sin x + 2y \cos x \frac{dy}{dx}$$

$$\cos(x+y) + \cos(x+y)\frac{dy}{dx} = -y^2 \sin x + 2y \cos x \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-y^2 \sin x - \cos(x+y)}{\cos(x+y) - 2y\cos x}$$

$$= \frac{y^2 \sin x + \cos(x+y)}{2y\cos x - \cos(x+y)}$$

Write an eq. of a tan to $x^3+y^3=6xy$ at $(3,3)$.

$$3x^2+3y^2\frac{dy}{dx}=6x\frac{dy}{dx}+6y$$

$$\frac{dy}{dx}=\frac{6y-3x^2}{3y^2-6x}$$

$$\left.\frac{dy}{dx}\right|_{(3,3)}=\frac{18-27}{27-18}=-1 \longrightarrow m_T=-1$$

$$\therefore\ y-3=-(x-3)$$

$$x^3 + y^3 = 6xy$$

$$3x^2 \frac{dx}{dt} + 3y^2 \frac{dy}{dt} = 6x \frac{dy}{dt} + 6y \frac{dx}{dt}$$

$$\sqrt{x} + \sqrt{y} = 4$$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$$

$$\frac{1}{2\sqrt{y}} \frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{\cancel{2}\sqrt{y}}{\cancel{2}\sqrt{x}}$$