The Derivative of the Natural Exponential Function

$\boxed{D_x\left[e^{f(x)}\right]}$

$x^n \quad nx^{n-1}$

$f(x)^n \quad nf(x)^{n-1} \cdot f'(x)$

$D_x\left[a^{f(x)}\right]$

$\sin x \quad \cos x$

$\sin f(x) \quad f'(x)\cos f(x)$

$D_x\left[\ln f(x)\right]$

$D_x\left[\log_a f(x)\right]$

$f(x) = e^x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x e^h - e^x}{h}$$

$$= \lim_{h \to 0} e^x \frac{e^h - 1}{h}$$

$$D_x[e^x] = e^x$$

$$= e^x \lim_{h \to 0} \frac{e^h - 1}{h}$$

$$D_x\left[e^{f(x)}\right] = e^{f(x)} \cdot f'(x) = f'(x)\, e^{f(x)}$$

$$f(x) = e^{x^3} \qquad f'(x) = 3x^2 e^{x^3}$$

$$g(x) = e^{\sin x} \qquad g'(x) = e^{\sin x} \cos x$$

$$h(x) = e^{\sin x^2} \qquad h'(x) = 2x\, e^{\sin x^2} \cos x^2$$

$$\lim_{x \to \infty} \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} = \lim_{x \to \infty} \frac{e^{2x} - \frac{1}{e^{2x}}}{e^{2x} + \frac{1}{e^{2x}}} = 1$$

$$\lim_{x \to -\infty} \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$$

$$= \lim_{x \to -\infty} \frac{e^{2x} - \frac{1}{e^{2x}}}{e^{2x} + \frac{1}{e^{2x}}}$$

$$= -1$$

$$f(x) = x^2 e^{x^2}$$

$$f'(x) = (x^2)(2x e^{x^2}) + (e^{x^2})(2x)$$

$$= 2x e^{x^2}(x^2 + 1)$$

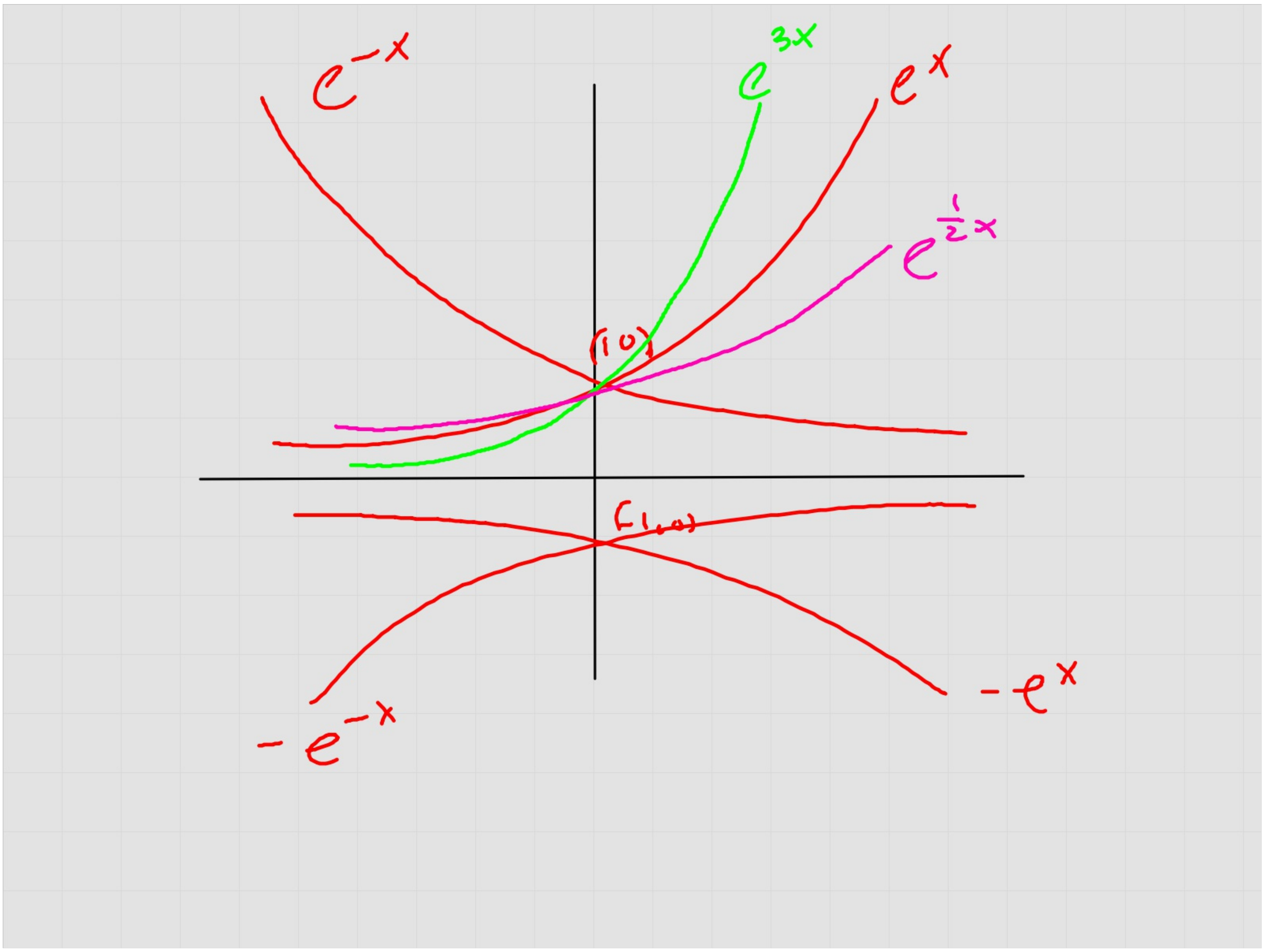
Write an eq. of a tan to $2e^{xy} = x + y$ @ $(0,2)$.

$$2e^{xy}\left(x\frac{dy}{dx} + y\right) = 1 + \frac{dy}{dx}$$

$$2xe^{xy}\frac{dy}{dx} + 2ye^{xy} = 1 + \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1 - 2ye^{xy}}{2xe^{xy} - 1} \rightarrow \left.\frac{dy}{dx}\right|_{(0,2)} = \frac{-3}{-1} = 3$$

$$\therefore\ y - 2 = 3(x - 0)$$



e^{-x}
e^{3x}
e^x
e^{\frac{1}{2}x}
(1 0)
(-1, 0)
-e^{-x}
-e^x