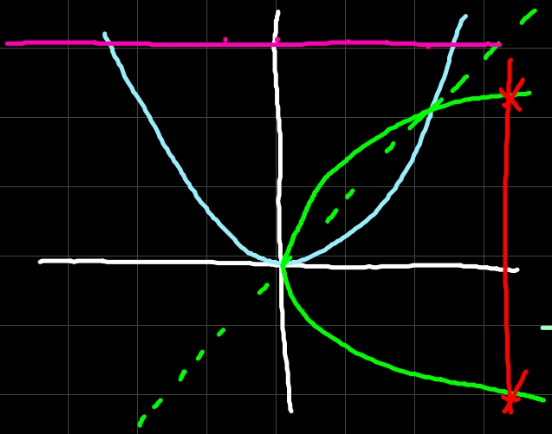
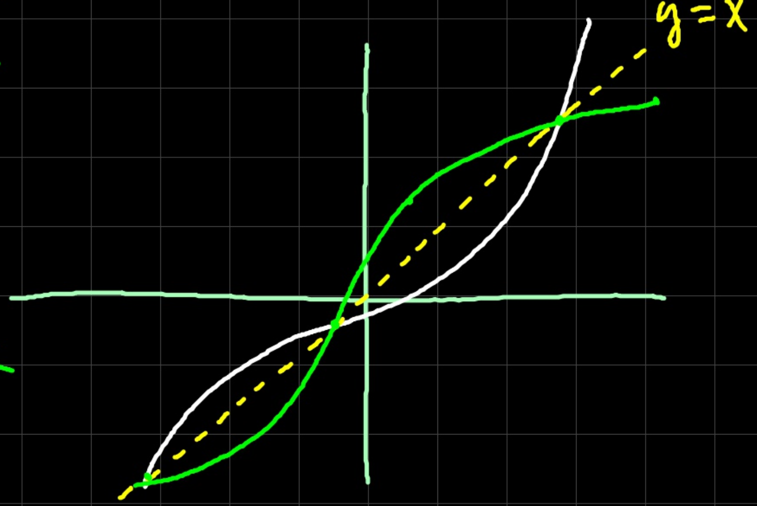
Inverse Functions

$y = x^2$ $x = y^2$

$f(x) = x^2$ no inverse





$f\ (2,3) \qquad g\ (3,2)$

$e^{\ln x} = x$

$f(g(3)) = f(2) = 3$

$\ln e^{x} = x$

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f and g are inverses iff

$f(g(x)) = x \ \forall x$ AND $g(f(x)) = x \ \forall x$

$f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

$\boxed{\text{Note: } x = f^{-1}(y) \quad y = f^{-1}(x)}$

Show that $f(x) = 3x-2$ and $g(x) = \frac{x+2}{3}$ are inverse.

$$f(g(x)) = f\left(\frac{x+2}{3}\right)$$
$$= 3\left(\frac{x+2}{3}\right) - 2$$
$$= x$$

$$g(f(x)) = g(3x-2)$$
$$= \frac{(3x-2)+2}{3}$$
$$= x$$

Since $f(g(x)) = x \ \forall x$ and $g(f(x)) = x \ \forall x$, f and g are inverse.

Find the inverse of $f(x) = \dfrac{x-2}{x+3}$.

Let $y = f(x)$

$$y = \frac{x-2}{x+3}$$

$$xy + 3y = x - 2$$

$$xy - x = -3y - 2$$

$$x(y-1) = -3y - 2$$

$$x = \frac{-3y-2}{y-1}$$

$$x = \frac{3y+2}{1-y}$$

$$f^{-1}(y) = \frac{3y+2}{1-y}$$

$$f^{-1}(x) = \frac{3x+2}{1-x}$$

Find the inven of $f(x) = 8x + 3$

→ Let $y = f(x)$

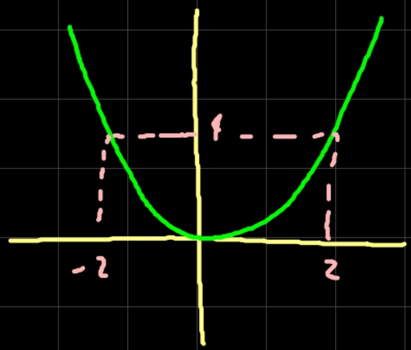
$y = 8x + 3$

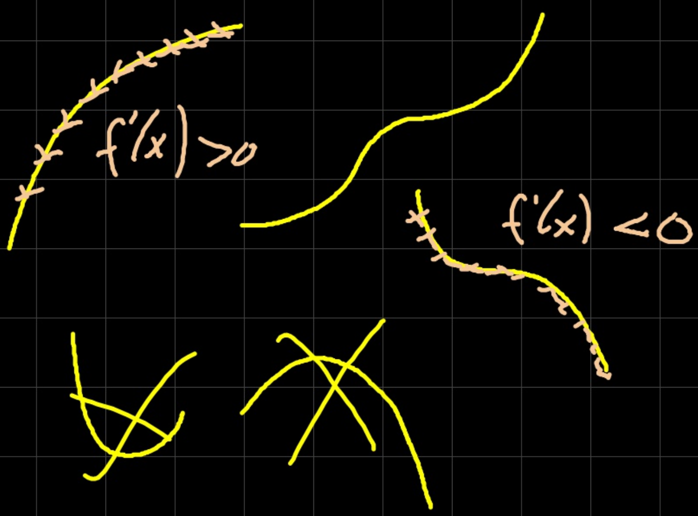
$x = \frac{y-3}{8}$

→ $f^{-1}(y) = \frac{y-3}{8}$

→ $f^{-1}(x) = \frac{x-3}{8}$

one-to-one "monotonic"





To show f has an inverse...

Show that $f'(x) \geq 0 \ \forall x$ in f

or $f'(x) \leq 0 \ \forall x$ in f

Show that $f(x) = \frac{x-3}{x+2}$ has an inverse.

$$f'(x) = \frac{5}{(x+2)^2}$$

f has an inverse because $f'(x) \geq 0 \ \forall x$ in f.

Determine if $f(x) = x^5 + 5x^3 + 2x - 4$ has an inverse.

$$f'(x) = 5x^4 + 15x^2 + 2$$

f has an inverse because $f'(x) \geq 0 \ \forall x$ in f.

Find f^{-1}

Let $y = f(x)$

$$y = x^5 + 5x^3 + 2x - 4$$

?

$f(x) = x^5 + 5x^3 + 2x - 4$

$(1, 4)$ on f

$(4, 1)$ on f^{-1}

$(f^{-1})'(4)$

$y - y_1 = m(x - x_1)$ f (c, d) tan at $x = c$

$y - d = f'(c)(x - c)$

$\frac{y - d}{f'(x)} + c = x$

$x - c = \frac{1}{f'(c)}(y - d)$

$y - c = \boxed{\frac{1}{f'(c)}}(x - d)$

eq tan to f^{-1} (d, c)

$$\boxed{\text{If } (c,d) \text{ is on } f \text{ then } (f^{-1})'(d) = \frac{1}{f'(c)}}$$

Given $f(x) = x^5 + 5x^3 + 2x - 4$, write eq of tan. to f^{-1} at $x = 4$.

$$x^5 + 5x^3 + 2x - 4 = 4 \rightarrow x = 1$$

Since $(1,4)$ is on f then $(f^{-1})'(4) = \frac{1}{f'(1)}$

$f'(x) = 5x^4 + 15x^2 + 2$

$f'(1) = 22 \rightarrow (f^{-1})'(4) = \frac{1}{22}$

We know that $(4,1)$ is on f^{-1}

$$\boxed{\therefore\ y - 1 = \frac{1}{22}(x - 4)}$$

Given that $f(2) = 5$ and $f'(2) = 7$

and $g(x) = f^{-1}(x)$, find $g'(5)$.

$(f^{-1})'(5)$

Since $(2, 5)$ on f

then $(f^{-1})'(5) = \frac{1}{f'(2)} = \frac{1}{7}$