Linearizations

(Tangent Line Approximations)

Write eq of tan to

$y = x^2 + 4x$ at $x = 1$

$y(1) = 5$

$\frac{dy}{dx} = 2x + 4$

$\left.\frac{dy}{dx}\right|_{x=1} = 6$

$y - 5 = 6(x-1)$

$y = 5 + 6(x-1)$

$y(1.1) \approx 5 + 6(1.1 - 1)$

Linearize $f(x) = x^2 + 4x$ at $x = 1$.

$f(1) = 5$

$f'(x) = 2x + 4$

$f'(1) = 6$

$L(x) = 5 + 6(x-1)$

$f(1.1) \approx L(1.1) = 5 + 6\left(\frac{1}{10}\right)$

$= 5.600$

$$y - y_1 = m(x - x_1) \qquad @\ x = a$$

$$y - f(a) = f'(a)(x-a)$$

$$\boxed{L(x) = f(a) + f'(a)(x-a)}$$

"linearization of f at $x=a$"

Linearize $f(x) = x^3 + 2x$ at $x = 5$.

$$f(5) = 125 + 10 = 135$$

$$f'(x) = 3x^2 + 2$$

$$f'(5) = 77$$

$$\therefore L(x) = 135 + 77(x - 5)$$

Use an approx. linearization of $f(x) = \sqrt{x}$ to est. $f(16.3)$.

Linearize f at $x = 16$

$f(16) = 4$

$f'(x) = \frac{1}{2\sqrt{x}} \longrightarrow f'(4) = \frac{1}{4}$

$L(x) = 4 + \frac{1}{4}(x - 16)$

$f(16.3) \approx L(16.3) = 4 + \frac{1}{4}\left(\frac{3}{10}\right) = 4 + \frac{3}{40} = 4.075$

Use an approp. linearization to est $\sqrt[3]{64.2}$.

Linearize $f(x) = \sqrt[3]{x}$ at $x = 64$

$$f(64) = 4$$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}} \longrightarrow f'(64) = \frac{1}{48}$$

$$L(x) = 4 + \frac{1}{48}(x - 64)$$

$$\sqrt[3]{64.2} \approx L(64.2) = 4 + \frac{1}{48}\left(\frac{2}{10}\right) = 4 + \frac{1}{240}$$