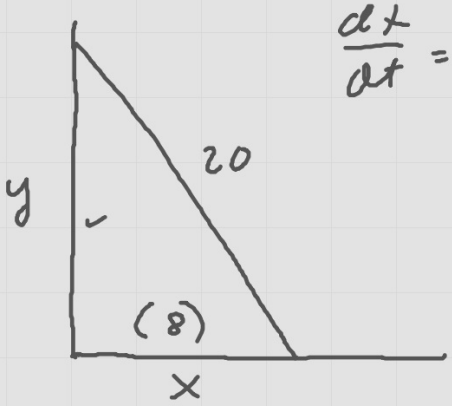
Related Rates



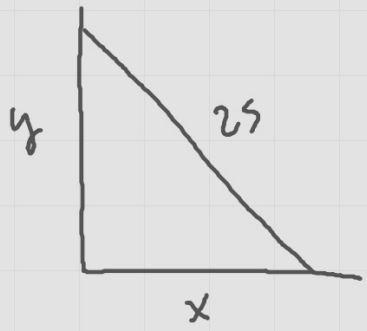
$\frac{dx}{dt} = 2$ ← Find $\frac{dy}{dt}$ when $x = 8$

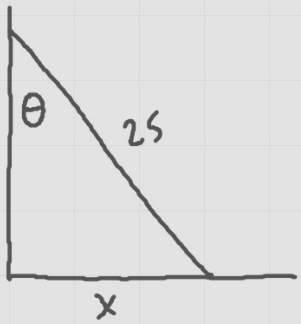
$$x^2 + y^2 = 20^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

- draw diagram / label & pull out known rates
- "find" statement
- write eq. that ties all known & unknowns together
  - known area/vol formulas
  - Pythagorean Thm
  - Similar Δ's
- diff. w/ respect to t
- plug n' chug

Ladder



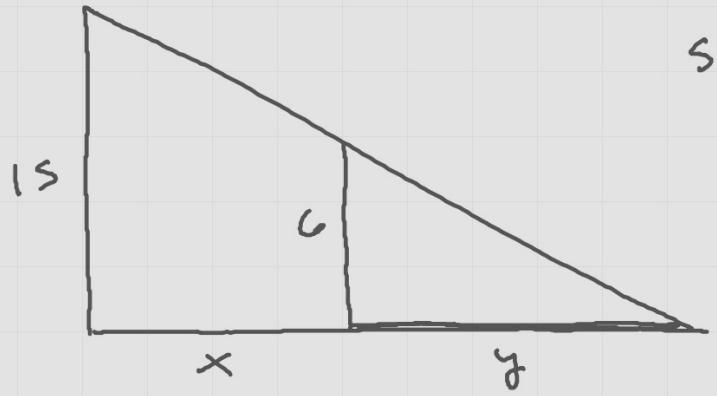


$$\sin\theta = \frac{x}{25}$$

$$\sin\theta = \frac{1}{25}x$$

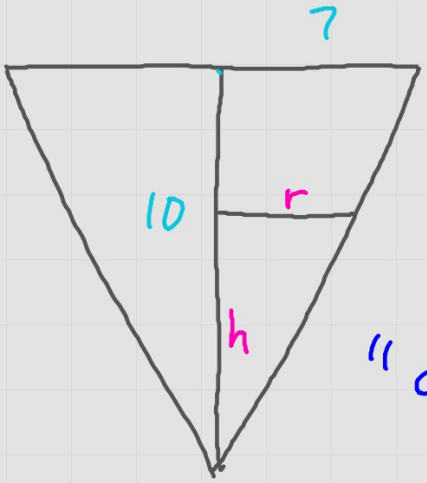
$$\cos\theta \frac{d\theta}{dt} = \frac{1}{25}\frac{dx}{dt}$$

Shadow



similar Δ's.

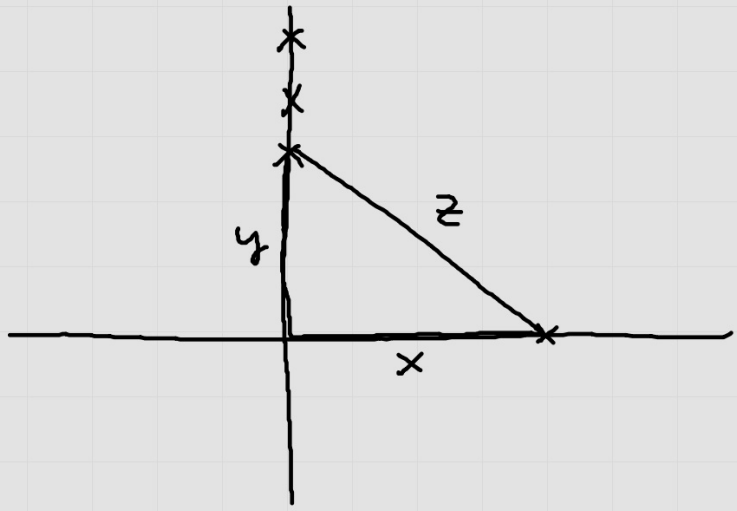
Cone



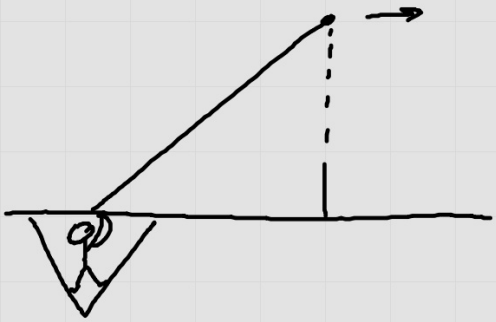
$V = \frac{1}{3}\pi r^2 h$

"dump the r"

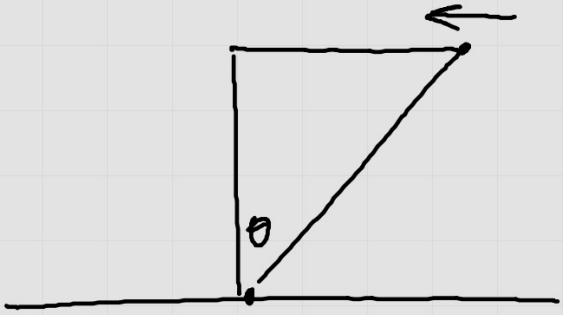
Intersect



Kite / Airplane



Radar / Lighthouse / Search light



① $\frac{dV}{dt} = 2$

Find $\frac{dr}{dt}$ when $r = .5$

$V = \frac{4}{3}\pi r^3$

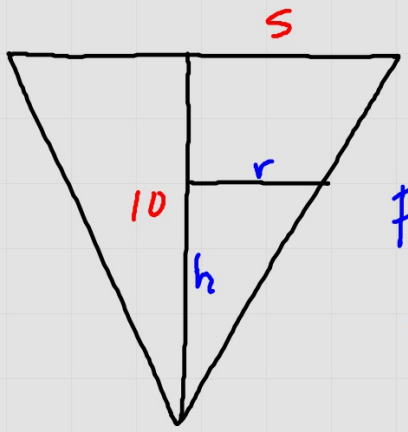
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$2 = 4\pi(.5)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = .637$

$\therefore$ the diameter is increasing at 1.273 in/sec.

(2)



$\frac{dV}{dt} = 3.14$

Find $\frac{dh}{dt}$ when $h = 7.5$

$\frac{10}{h} = \frac{5}{r}$

$10r = 5h$

$r = \frac{h}{2}$

$r^2 = \frac{h^2}{4}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \frac{h^2}{4} h$

$V = \frac{\pi}{12} h^3$

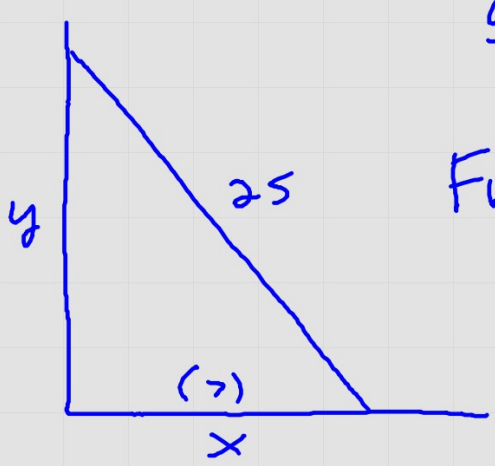
$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$3.14 = \frac{\pi}{4}(7.5)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = .071$

∴ water level is rising at .071 ft/min

(3)



$\frac{dy}{dt} = -1$

Find $\frac{dx}{dt}$ when $x = 7$

$x^2 + y^2 = 25^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

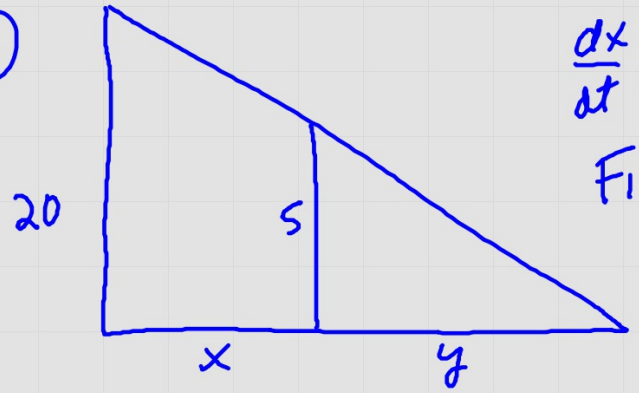
$y = \sqrt{625 - 49}$
$= 24$

$(7)\frac{dx}{dt} + 24(-1) = 0$

$\frac{dy}{dt} = \frac{24}{7}$

∴ bottom moving away from wall at $\frac{24}{7}$ ft/min.

④



$\frac{dx}{dt} = -6$

Find $\frac{dy}{dt}$

$\frac{20}{x+y} = \frac{5}{y}$

$5x + 5y = 20y$

$5x = 15y$

$x = 3y$

$x = 3y$

$\frac{dx}{dt} = 3\frac{dy}{dt}$

$\frac{-6}{3} = \frac{dy}{dt}$

$\frac{dy}{dt} = -2$

∴ shadow getting shorter at 2 ft/sec.

∴ tip is moving toward light at 8 ft/sec.