Implicit Differentiation

Explicitly defined

$y = x^2 + 3x - 1$

$f(x) = x^3 \sin x$

Find $\frac{dy}{dx}$

Implicitly defined

$x^2 + y^2 = 25$

$x^3 + y^3 = 6xy$

Find $\frac{dy}{dx}$

$\frac{dx}{dy}$

$$D_x\left[f(x)^3\right] = 3f(x)^2 \, f'(x)$$

$$y = x^5$$

$$\frac{dy}{dx} = 5x^4$$

$$D_x\left[y^3\right] = 3y^2 \frac{dy}{dx}$$

$x^3 + y^3 = 15$ find $\frac{dy}{dx}$. $\frac{dx}{dy}$

$$3x^2 + 3y^2 \frac{dy}{dx} = 0$$

$$3x^2 \frac{dx}{dy} + 3y^2 = 0$$

$$\frac{dy}{dx} = -\frac{3x^2}{3y^2}$$

$$= -\frac{x^2}{y^2}$$

$$\frac{dx}{dy} = -\frac{y^2}{x^2}$$

$x^2 + y^2 = 25$ diff. w/ respect to t.

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$D_t \left[w^3 \right] = 3w^2 \frac{dw}{dt}$$

$y^5 - 3x^2y^2 + 5x^4 = 12$ find $\frac{dy}{dx}$.

$5y^4 \frac{dy}{dx} - \left[3x^2(2y)\frac{dy}{dx} + 6xy^2 \right] + 20x^3 = 0$

$5y^4 \frac{dy}{dx} - 6x^2y\frac{dy}{dx} - 6xy^2 + 20x^3 = 0$

$$\frac{dy}{dx} = \frac{6xy^2 - 20x^3}{5y^4 - 6x^2y}$$

$\sqrt{x} + \sqrt{y} = 4$ find $\frac{dy}{dx}$.

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\frac{1}{2\sqrt{y}}\frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$\sin(x+y) = y^2 \cos x$ find $\frac{dy}{dx}$.

$$\cos(x+y)\left[1+\frac{dy}{dx}\right] = -y^2\sin x + 2y\cos x\,\frac{dy}{dx}$$

$$\cos(x+y) + \cos(x+y)\frac{dy}{dx} = -y^2\sin x + 2y\cos x\,\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-y^2\sin x - \cos(x+y)}{\cos(x+y) - 2y\cos x}$$

$$x^3 + y^3 = xy$$

$$3x^2 \cancel{\frac{dx}{dx}} + 3y^2 \frac{dy}{dx} = x \frac{dy}{dx} + y \cancel{\frac{dx}{dx}}$$

Write eq. of tan to $x^3+y^3=6xy$ at $(3,3)$.

$$3x^2+3y^2\frac{dy}{dx}=6x\frac{dy}{dx}+6y$$

$$\frac{dy}{dx}=\frac{6y-3x^2}{3y^2-6x}$$

$$\left.\frac{dy}{dx}\right|_{(3,3)}=\frac{18-27}{27-18}=-1 \rightarrow m_T=-1$$

$$\therefore y-3=-(x-3)$$