Derivative Test

- 9 derivatives
- 2 equations of tangents (Cat I and II)
- 1 differentiability
- 1 definition of derivative
- 1 Always, Sometimes, Never about continuity and differentiability

- 14 @ 5: 70 points

Like last test--the following problems are NOT a comprehensive review--they are primarily to demonstrate *presentation* of solutions

1. Find the derivative of $f(x) = \dfrac{x^3 - 8x^2 + 2}{\sqrt{x}}$.

$$f'(x) = \frac{\sqrt{x}\,(3x^2 - 16x) - (x^3 - 8x^2 + 2)\,\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$$

2. Find the derivative of $f(x) = -\sin(4x - 9)$.

$$f'(x) = (-\cos(4x-9))(4)$$

OR

$$f'(x) = -4\cos(4x-9)$$

3. Find the derivative of $f(x) = \sec^2 3x^4$.

$$f'(x) = \left[2\sec 3x^4\right]\left[\sec 3x^4 \tan 3x^4\right]\left[12x^3\right]$$

4. Find the derivative of $f(x) = \tan 2x \cos 5x$.

$$f'(x) = (\tan 2x)(-\sin 5x)(5) + (\cos 5x)(\sec^2 2x)(2)$$

5. Find the derivative of $f(x) = \dfrac{2x^3 + 7x}{x^2 - 3x}$.

$$f'(x) = \frac{(x^2-3x)(6x^2+7)-(2x^3+7x)(2x-3)}{(x^2-3x)^2}$$

6. Find the derivative of $g(x) = -2x^3\sqrt{x^3 - 1}$.

$$g'(x) = \left(-2x^3\right)\frac{3x^2}{2\sqrt{x^3-1}} + \left(\sqrt{x^3-1}\right)\left(-6x^2\right)$$

$$h(x) = \sqrt{5 + \sqrt{x}}$$

$$h'(x) = \frac{\frac{1}{2\sqrt{x}}}{2\sqrt{5+\sqrt{x}}}$$

7. Find an equation of the tangent to the curve $y = 3x^2 - 4x + 1$ at $x = 2$.

$y(2) = 12 - 8 + 1 = 5 \longrightarrow (2, 5)$.

$\frac{dy}{dx} = 6x - 4$

$\left.\frac{dy}{dx}\right|_{x=2} = 8 \longrightarrow m_T = 8$

$\therefore y - 5 = 8(x - 2)$

8. Find an equation of the tangent to $y = x^4 - 1$ that is parallel to $64x - 2y + 7 = 0$.

Slope of given line is $32 \rightarrow m_T = 32$

$$\frac{dy}{dx} = 4x^3$$

$$4x^3 = 32 \rightarrow x = 2 \rightarrow y = 15 \rightarrow (2, 15)$$

$$\therefore \; y - 15 = 32(x - 2).$$

9. Given $f(x) = \cos x$, find $f^{219}(x)$.

$$4 \overline{)219} \quad 54 R 3$$

$f^{219}(x) = f'''(x)$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f'''(x) = \sin x$

$\therefore f^{219}(x) = \sin x$.

10. At what values of x is the following function NOT differentiable? Make sure you completely justify your answer.

$$f(x) = \begin{cases} x+5 & \text{if } x \le -1 \\ 4x^2 & \text{if } -1 < x < 1 \\ 3-x & \text{if } x \ge 1 \end{cases}$$

$1 \quad 1$

$8x \quad -8 \quad 8$

$-1 \quad -1$

$$f'(x) = \begin{cases} 1 & x < -1 \\ 8x & -1 < x < 1 \\ -1 & x > 1 \end{cases}$$

Since $f'_+(-1) = -8$ but $f'_-(-1) = 1 \rightarrow f'(-1) \nexists$

$\therefore$ f not diff at $x = -1$

Since $f'_+(1) = -1$ but $f'_-(1) = 8 \rightarrow f'(1) \nexists$

$\therefore$ f not diff at $x = 1$.

11. Given $g(x) = \sqrt{\dfrac{x-1}{4-x}}$, find $g'(x)$.

$$g'(x) = \frac{\dfrac{(4-x)(1) - (x-1)(-1)}{(4-x)^2}}{2\sqrt{\dfrac{x-1}{4-x}}}$$

12. Suppose $h(x) = g(f(x))$ and $f(2) = 6$, $f'(2) = -3$, $g(2) = 4$ and $g'(6) = 8$. Find $h'(2)$.

$$h'(x) = g'(f(x)) \cdot f'(x)$$

$$h'(2) = g'(f(2)) \cdot f'(2)$$

$$= g'(6)(-3)$$

$$= (8)(-3)$$

$$= -24$$

Given $f(x) = x^2 + x + 1$, find $f'(x)$ using the definition of derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + (x+h) + 1 - (x^2 + x + 1)}{h}$$

$$= 2x + 1$$