The Derivative of the Natural Exponential Function

$$\boxed{D_x\left[e^{f(x)}\right]}$$

$$D_x\left[\sin x\right] = \cos x \checkmark$$

$$D_x\left[\sin f(x)\right] = f'(x)\cos f(x)$$

$$D_x\left[a^{f(x)}\right]$$

$$D_x\left[\ln f(x)\right]$$

$$D_x\left[\log_a f(x)\right]$$

$f(x) = e^x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x e^h - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x(e^h - 1)}{h}$$

$$= e^x \boxed{\lim_{h \to 0} \frac{e^h - 1}{h}}$$

$$= e^x$$

$$D_x\left[e^x\right] = e^x$$

$$D_x\left[e^{f(x)}\right] = e^{f(x)} f'(x) = f'(x)e^{f(x)}$$

$f(x) = e^{5x}$

$f'(x) = 5e^{5x}$

$g(x) = e^{\sin x}$

$g'(x) = e^{\sin x} \cos x$

$$f(x) = e^{\sin 3x}$$

$$f'(x) = 3e^{\sin 3x} \cos 3x$$

$$g(x) = x^2 e^{x^2}$$

$$g'(x) = (x^2)(2x e^{x^2}) + (e^{x^2})(2x)$$

$$= 2x e^{x^2}(x^2 + 1)$$

$$2e^{xy} = x + y$$

$$2e^{xy}\left(x\frac{dy}{dx} + y\right) = 1 + \frac{dy}{dx}$$

$$2xe^{xy}\frac{dy}{dx} + 2ye^{xy} = 1 + \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1 - 2ye^{xy}}{2xe^{xy} - 1}$$

$$\lim_{x \to \infty} \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} = \lim_{x \to \infty} \frac{e^{2x} - \frac{1}{e^{2x}}}{e^{2x} + \frac{1}{e^{2x}}} = 1$$

$$\lim_{x \to -\infty} \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} = \lim_{x \to -\infty} \frac{e^{2x} - \frac{1}{e^{2x}}}{e^{2x} + \frac{1}{e^{2x}}} = -1$$

$$f(x) = \sqrt{e^{2x} + 2x}$$

$$f'(x) = \frac{2e^{2x} + 2}{2\sqrt{e^{2x} + 2x}}$$

$$= \frac{e^{2x} + 1}{\sqrt{e^{2x} + 2x}}$$