Inverse Functions

f and g are inverses iff

$f(g(x)) = x$ and $g(f(x)) = x$ $\forall x$ in f.

$f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

Show that $f(x) = 3x-2$ and $g(x) = \frac{x+2}{3}$ are inverse.

$$f(g(x)) = f\left(\frac{x+2}{3}\right)$$
$$= 3\left(\frac{x+2}{3}\right) - 2$$
$$= x$$

$$g(f(x)) = g(3x-2)$$
$$= \frac{(3x-2)+2}{3}$$
$$= x$$

Since $f(g(x)) = x$ and $g(f(x)) = x$ $\forall x$, f and g are inverses.

Notation

$$x = f^{-1}(y) \quad \text{and} \quad y = f^{-1}(x)$$

Find the inverse of $f(x) = 7x + 5$.

let $y = f(x)$

$$y = 7x + 5$$

$$x = \frac{y-5}{7}$$

$$f^{-1}(y) = \frac{y-5}{7}$$

$$f^{-1}(x) = \frac{x-5}{7}$$

Find the inverse of $f(x) = \dfrac{x+2}{x-3}$.

Let $y = f(x)$

$$y = \frac{x+2}{x-3}$$

$$y(x-3) = x+2$$

$$xy - 3y = x+2$$

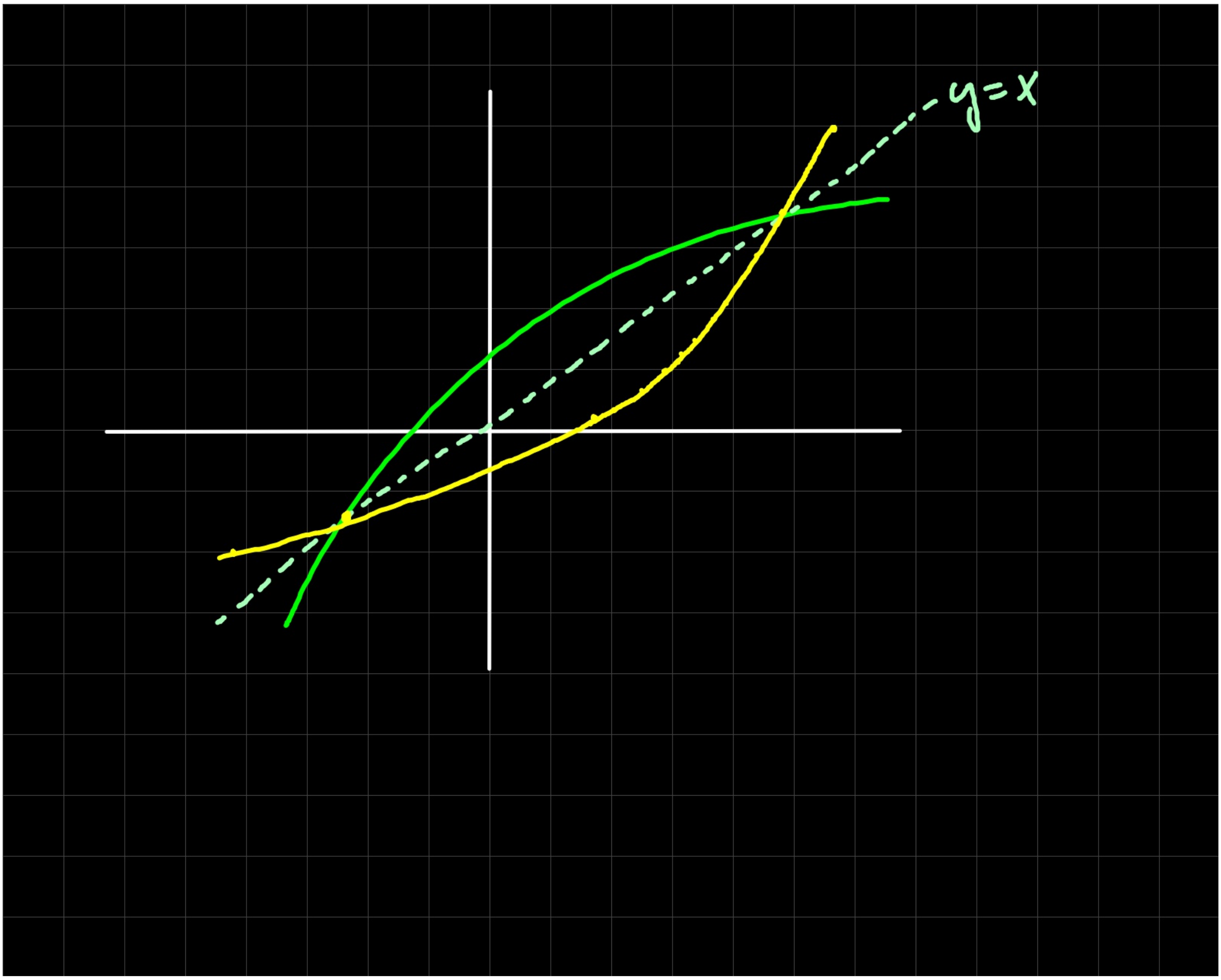
$$xy - x = 3y+2$$

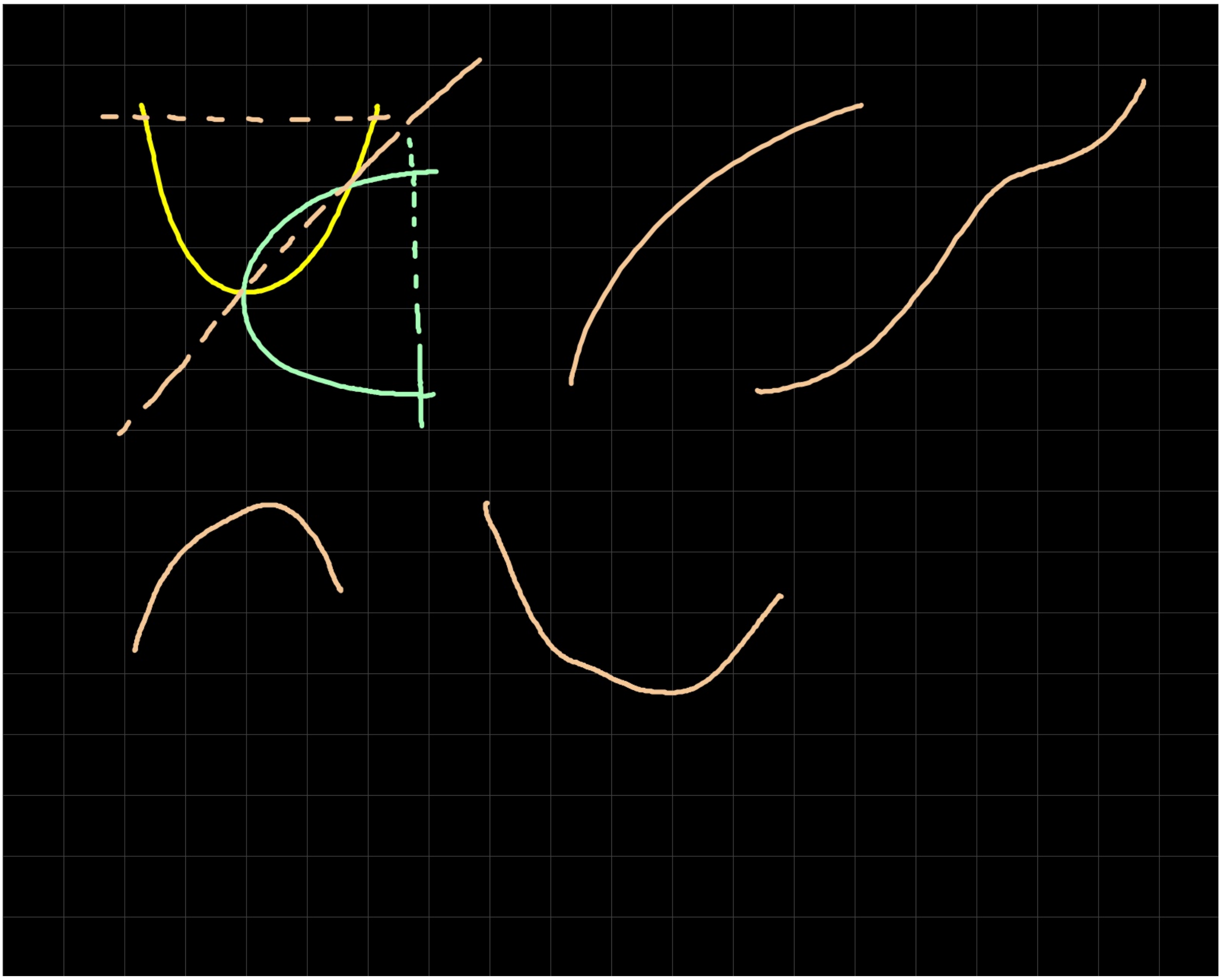
$$x(y-1) = 3y+2$$

$$x = \frac{3y+2}{y-1}$$

$$f^{-1}(y) = \frac{3y+2}{y-1}$$

$$\therefore f^{-1}(x) = \frac{3x+2}{x-1}$$


y=x



To have an inverse a function must be one-to-one. (monotonic)

If $f'(x) \geq 0 \quad \forall x$ in f

or

$f'(x) \leq 0 \quad \forall x$ in f

then f has an inverse.

Determine if $f(x) = \frac{x}{x+2}$ has an inverse.

$$f'(x) = \frac{(x+2) - x}{(x+2)^2} = \frac{2}{(x+2)^2}$$

Since $f'(x) \geq 0 \ \forall x$ in f

f has an inverse.

Determine if $f(x) = x^5 + 5x^3 + 2x - 4$ has an inverse.

$f'(x) = 5x^4 + 15x^2 + 2$

Since $f'(x) \geq 0 \ \forall x$ in f, f has an inverse.

Find it.

Let $y = f(x)$

$y = x^5 + 5x^3 + 2x - 4$

$y - y_1 = m(x - x_1)$ tan f at (c, d)

$y - d = f'(c)(x - c)$ tan f

$$\frac{y-d}{f'(c)} = x - c$$

$$x - c = \frac{1}{f'(c)}(y - d)$$

$$x = \frac{1}{f'(c)}(y - d) + c$$

$f^{-1}(x) = \frac{1}{f'(c)}(x - d) + c$ tan to f^{-1}

If (c,d) is on f then

$$\boxed{(f^{-1})'(d) = \frac{1}{f'(c)}}$$

Write eq of tan to the inverse of $f(x) = x^5 + 5x^3 + 2x - 4$ at $x = 4$.

$x^5 + 5x^3 + 2x - 4 = 4 \rightarrow x = 1.$

Since $(1,4)$ is on f then $(f^{-1})'(4) = \frac{1}{f'(1)}$

$f'(x) = 5x^4 + 15x^2 + 2$

$f'(1) = 22 \rightarrow (f^{-1})'(4) = \frac{1}{22}$

Since $(1,4)$ on $f \rightarrow (4,1)$ on f^{-1}

$y - 1 = \frac{1}{22}(x - 4)$

Given that $f(2) = 5$ and $f'(2) = 7$

and $g(x) = f^{-1}(x)$ find $g'(5)$.

$(2, 5)$ on f

$$(f^{-1})'(5) = \frac{1}{f'(2)}$$

$$g'(5) = \frac{1}{f'(2)}$$

$$= \frac{1}{7}$$