Linearizations

(Tangent Line Approximations)

$$y - y_1 = m(x - x_1) \qquad x = a$$

$$y - f(a) = f'(a)(x - a)$$

$$y = f(a) + f'(a)(x - a)$$

$$\boxed{L(x) = f(a) + f'(a)(x - a)}$$

Write eq of tan to
$y = x^2 + 4x$ at
$x = 1$ & use it to approx
$y(1.2)$

$y(1) = 5 \rightarrow (1, 5)$

$$\frac{dy}{dx} = 2x + 4$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 6$$

$\therefore y - 5 = 6(x - 1)$

$y = 5 + 6(x - 1)$

$y(1.2) = 5 + 6(.2) = 6.2$

Linearize $f(x) = x^2 + 4x$ at $x = 1$.
and use it to approx $f(1.2)$.

$f(1) = 5 \rightarrow (1, 5)$

$f'(x) = 2x + 4$

$f'(1) = 6 \rightarrow m_T = 6$

$L(x) = 5 + 6(x - 1)$

$L(1.2) = 5 + 6(.2) = 6.2$

$\therefore f(1.2) \approx 6.2$

Linearize $f(x) = \sin x$ at $x = 0$.

$$f(0) = 0$$
$$f'(x) = \cos x$$
$$f'(0) = 1$$
$$\therefore L(x) = 0 + 1(x - 0)$$
$$L(x) = x$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Linearize $f(x)=\sqrt{x}$ at $x=25$

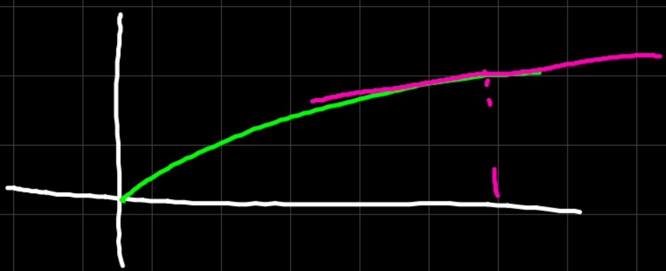
$\sqrt{25.2}\approx 5.020$

$f(25)=5$

$f'(x)=\frac{1}{2\sqrt{x}}$

$f'(25)=\frac{1}{10}$

$L(x)=5+\frac{1}{10}(x-25)$



$L(25.2)=5+\frac{1}{10}(.2)$

$=5+\frac{1}{50}$

$=5.02$

$\sqrt{30}\approx 5.477$

$L(30)=5+\frac{1}{10}(5)$

$=5.500$

Use an appropriate linearization of $f(x) = \sqrt{x}$ to est. $f(16.3)$.

$\frac{1}{8} \cdot \frac{3}{10}$

Linearize f at $x = 16$

$f(16) = 4$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(16) = \frac{1}{8}$

$L(x) = 4 + \frac{1}{8}(x - 16)$

$f(16.3) \approx L(16.3) = 4 + \frac{1}{8}(.3) = 4 + \frac{3}{80}$

Use an appro. linearization to est $\sqrt[3]{64.1}$.

Linearize $f(x) = \sqrt[3]{x}$ at $x = 64$

$f(64) = 4$

$f'(x) = \dfrac{1}{3\sqrt[3]{x^2}}$

$f'(64) = \dfrac{1}{3 \cdot 16} = \dfrac{1}{48}$

$L(x) = 4 + \frac{1}{48}(x - 64)$

$\sqrt[3]{64.1} \approx L(64.1) = 4 + \frac{1}{48}(.1) = 4 + \frac{1}{480}$