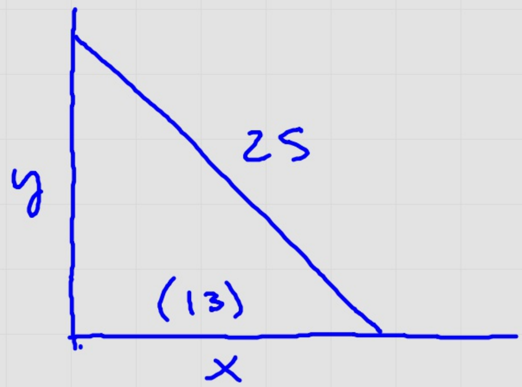
Related Rates

Applications of implicit diff.

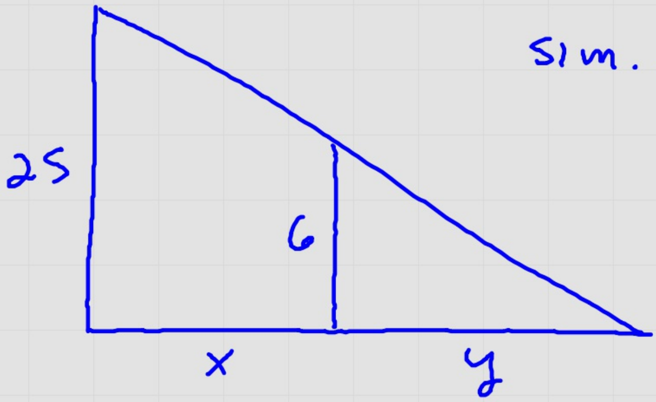
- draw a picture if possible
- pull info. out of problem
- "find" statement
- equation ties unknown in problem.
  - known area / vol.
  - P.T.
  - similar Δ's
- diff w/ respect to t.
- plug & chug

LADDER PROBLEM

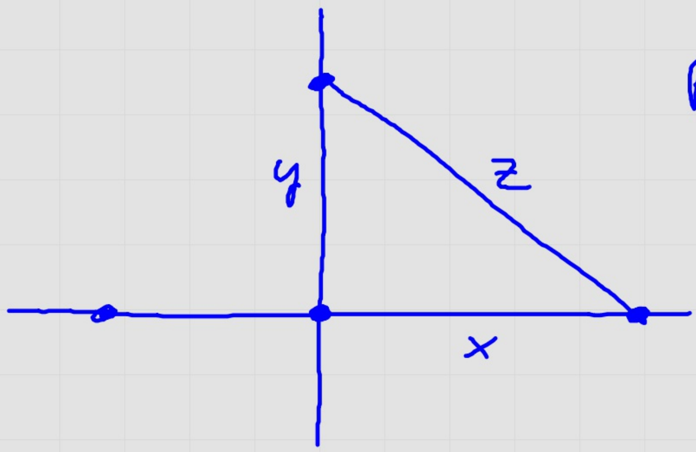


SHADOW PROBLEM

sim. Δ's

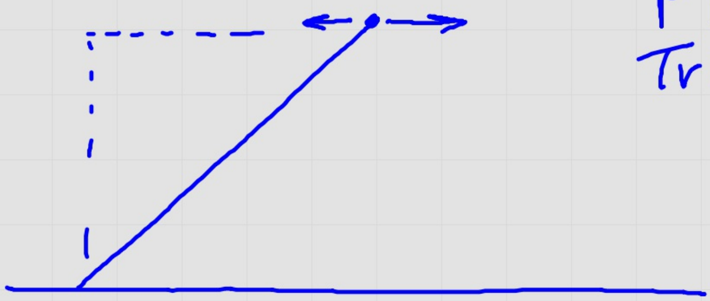


INTERSECTION PROBLEM



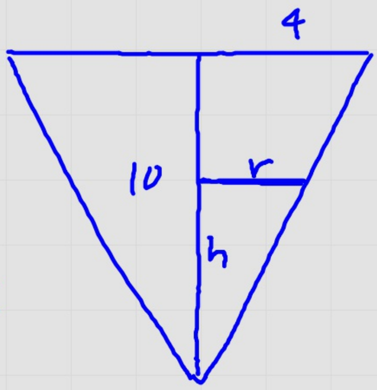
P.T.

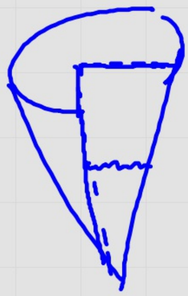
KITE / AIRPLANE / SEARCHLIGH



PT
Trig

Inverted Right Circular Cone





$$\frac{10}{h} = \frac{4}{r}$$

$$10r = 4h$$

$$r = \frac{2h}{5}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{4h^2}{25}\right)h$$

#1

$$\frac{dV}{dt} = 2$$

Find $\frac{dr}{dt}$ when $r = .5$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$2 = 4\pi(.5)^2 \frac{dr}{dt}$$

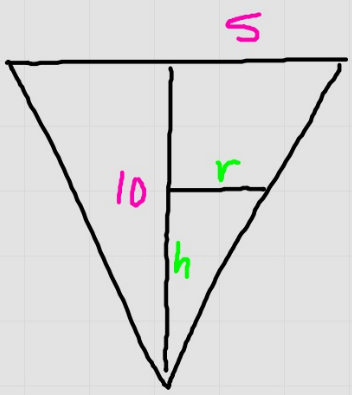
solve(2=4*π*(.5)^2·x, x)

$$\frac{dr}{dt} = .637$$

∴ the diameter is increasing at 1.273 in/sec.

#2

$$\frac{dV}{dt} = 3.14$$



$$\frac{10}{h} = \frac{5}{r}$$

$$10r = 5h$$

$$r = \frac{h}{2}$$

Find $\frac{dh}{dt}$ when $h = 7.5$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \frac{h^2}{4} h$$

$$V = \frac{\pi}{12} h^3$$

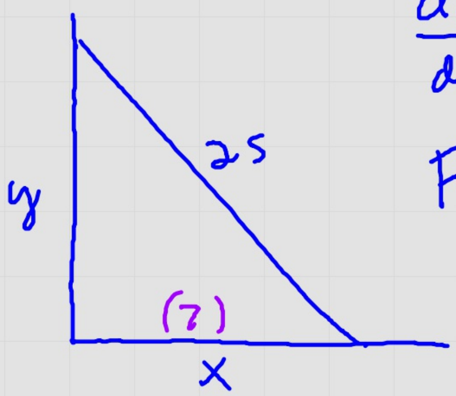
$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$3.14 = \frac{\pi}{4}(7.5)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = .071$$

$\therefore$ ht is incrasing at .071 ft/min

#3



$$\frac{dy}{dt} = -1$$

Find $\frac{dx}{dt}$ when $x = 7$

$y = \sqrt{25^2 - 49}$

$y = 24$

$$x^2 + y^2 = 25^2$$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$$

$$(7)\frac{dx}{dt} + (24)(-1) = 0$$

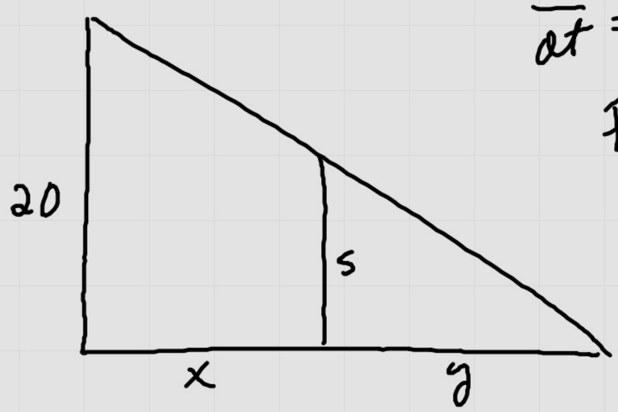
$$\frac{dx}{dt} = \frac{24}{7}$$

$\therefore$ bottom moving away from wall at $\frac{24}{7}$ ft/min

#4

$$\frac{dx}{dt} = -6$$

Find $\frac{dy}{dt}$



$$\frac{20}{x+y} = \frac{s}{y}$$

$$5x + 5y = 20y$$

$$5x = 15y$$

$$x = 3y$$

$$\frac{dx}{dt} = 3\frac{dy}{dt}$$

$$-6 = 3\frac{dy}{dt}$$

$$\frac{dy}{dt} = -2$$

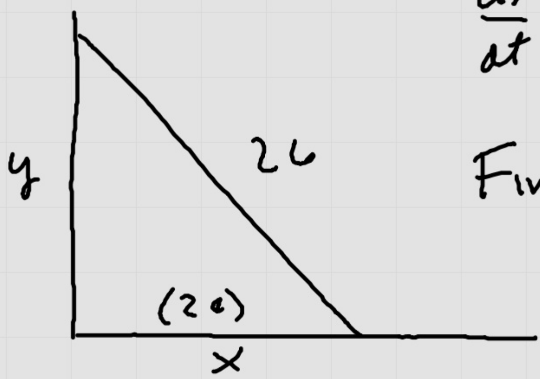
$\therefore$ shadow shorter at 2 ft/sec.

$\therefore$ tip moving at 8 ft/sec toward wall.

#3

$\frac{dx}{dt} = 2$

Find $\frac{dy}{dt}$ when $x = 24$



$y = \sqrt{26^2 - 24^2}$
$= 10$

$x^2 + y^2 = 26^2$

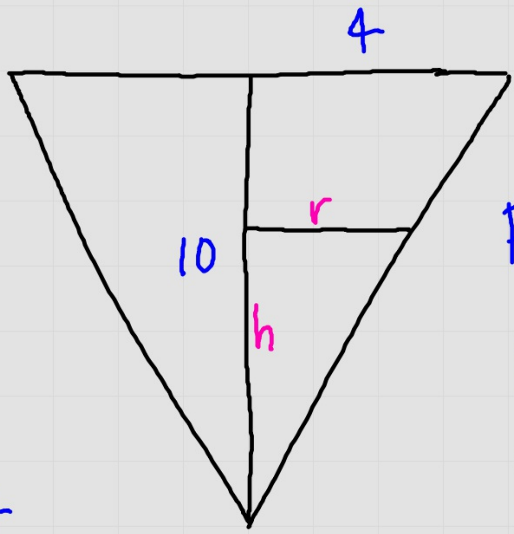
$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$

$24(2) + 10\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-48}{10}$

$\therefore$ top down wall at 4.800 ft/sec.

#6



$$\frac{dV}{dt} = 9$$

Find $\frac{dh}{dt}$ when $h = 5$

$$\frac{10}{h} = \frac{4}{r}$$

$$10r = 4h$$

$$r = \frac{2h}{5}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \frac{4h^2}{25} h$$

$$V = \frac{4\pi}{75} h^3$$

$$\frac{dV}{dt} = \frac{4\pi}{25} h^2 \frac{dh}{dt}$$

$$9 = \frac{4\pi}{\cancel{25}} (\cancel{25}) \frac{dh}{dt}$$

$$\frac{dh}{dt} = .716$$

$\therefore$ water level rising at .716 ft/min