Implicit Differentiation

Explicitly defined	Implicitly defined
$y = x^3 - x$	$x^2 + y^2 = 25$
$f(x) = x^2 \sin x$	$xy^3 - y \sin x = x^2$

$$D_x\left[f(x)^2\right] = 2f(x)\,f'(x)$$

$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}$$

$$D_x\left[y^3\right] = 3y^2\,\frac{dy}{dx}$$

$$D_x\left[x^2 f(x)^3\right] = (x^2)\left(3f(x)^2 f'(x)\right) + f(x)^3(2x)$$

$$D_x\left[x^2 y^3\right] = (x^2)\left(3y^2 \frac{dy}{dx}\right) + (y^3)(2x)$$

$$= 3x^2y^2\frac{dy}{dx} + 2xy^3$$

$x^3 + y^2 = 15$ find $\frac{dy}{dx}$.

$$3x^2 + 2y\frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = -\frac{3x^2}{2y}}$$

$$x^3 + y^3 = 6xy$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 6x \frac{dy}{dx} + 6y$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

$$3y^2 \frac{dy}{dx} - 6x \frac{dy}{dx}$$

$$\frac{dy}{dx}\left[3y^2 - 6x\right]$$

$$\sin(x+y) = y^2 \cos x$$

$$\cos(x+y)\left[1 + \frac{dy}{dx}\right] = -y^2 \sin x + 2y \cos x \frac{dy}{dx}$$

$$\cos(x+y) + \cos(x+y)\frac{dy}{dx} = -y^2 \sin x + 2y \cos x \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-y^2 \sin x - \cos(x+y)}{\cos(x+y) - 2y \cos x}$$

$$\sqrt{x} + \sqrt{y} = 4$$

$$\rightarrow \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$$

$$\frac{1}{2\sqrt{y}} \frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{2\sqrt{y}}{2\sqrt{x}}$$

$$= -\frac{\sqrt{y}}{\sqrt{x}}$$

$$D_x[\sec y] = \sec y \tan y \frac{dy}{dx}$$

$$D_y[\sec y] = \sec y \tan y$$

$$D_t[x^2] = 2x \frac{dx}{dt}$$

$$D_w[g^7] = 7g^6 \frac{dg}{dw}$$

Write eq n tan to $x^3+y^3=6xy$ at $(3,3)$.

$$3x^2+3y^2\frac{dy}{dx}=6x\frac{dy}{dx}+6y$$

$$\frac{dy}{dx}=\frac{6y-3x^2}{3y^2-6x}$$

$$\left.\frac{dy}{dx}\right|_{(3,3)}=\frac{18-27}{27-18}=-1\longrightarrow m_T=-1$$

$$\therefore y-3=-(x-3)$$