Derivative Test

- 9 derivatives
- 2 equations of tangents (Cat I and II)
- 1 differentiability
- 1 definition of derivative
- 1 Always, Sometimes, Never about continuity and differentiability

- 14 @ 5: 70 points

Like last test--the following problems are NOT a comprehensive review--they are primarily to demonstrate *presentation* of solutions

1. Find the derivative of $f(x) = \dfrac{x^3 - 8x^2 + 2}{\sqrt{x}}$.

$$f'(x) = \frac{(\sqrt{x})(3x^2 - 16x) - (x^3 - 8x^2 + 2)\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$$

2. Find the derivative of $f(x) = -\sin(4x - 9)$.

$f'(x) = -4\cos(4x-9)$

OR

$f'(x) = \left(-\cos(4x-9)\right)(4)$

3. Find the derivative of $f(x) = \sec^2 3x^4$.

$$f'(x) = \left[2 \sec 3x^4\right]\left[\sec 3x^4 \tan 3x^4\right](12x^3)$$

4. Find the derivative of $f(x) = \tan 2x \cos 5x$.

$$f'(x) = (\tan 2x)((-\sin 5x)(5)) + (\cos 5x)(\sec^2 2x)(2)$$

5. Find the derivative of $f(x) = \dfrac{2x^3 + 7x}{x^2 - 3x}$.

$$f'(x) = \frac{(x^2-3x)(6x^2+7) - (2x^3+7x)(2x-3)}{(x^2-3x)^2}$$

6. Find the derivative of $g(x) = -2x^3\sqrt{x^3 - 1}$.

$$g'(x) = (-2x^3)\frac{3x^2}{2\sqrt{x^3-1}} + \left(\sqrt{x^3-1}\right)(-6x^2)$$

7. Find an equation of the tangent to the curve $y = 3x^2 - 4x + 1$ at $x = 2$.

$y(2) = 12 - 8 + 1 = 5 \rightarrow (2, 5)$

$\frac{dy}{dx} = 6x - 4$

$\left.\frac{dy}{dx}\right|_{x=2} = 8 \rightarrow m_T = 8$

$\therefore y - 5 = 8(x - 2)$

8. Find an equation of the tangent to $y = x^4 - 1$ that is parallel to $64x - 2y + 7 = 0$.

Slope of given line is 32. $\rightarrow m_T = 32$

$$\frac{dy}{dx} = 4x^3$$

$$\therefore 4x^3 = 32 \rightarrow x = 2 \rightarrow y = 15 \rightarrow (2, 15)$$

$$\therefore y - 15 = 32(x - 2)$$

9. Given $f(x) = \cos x$, find $f^{219}(x)$.

$$4 \overline{)219} \quad 54 \text{ R } 3$$

$$f^{219}(x) = f'''(x)$$

$$f'(x) = -\sin x$$

$$f''(x) = -\cos x$$

$$f'''(x) = \sin x$$

$$\therefore f^{219}(x) = \sin x$$

10. At what values of x is the following function NOT differentiable? Make sure you completely justify your answer.

$$f(x) = \begin{cases} x+5 & \text{if } x \le -1 \\ 4x^2 & \text{if } -1 < x < 1 \\ 3-x & \text{if } x \ge 1 \end{cases}$$

$$\begin{array}{ccc} 1 & 1 & \\ 8x & -8 & 8 \\ -1 & & -1 \end{array}$$

$$f'(x) = \begin{cases} 1 & x < -1 \\ 8x & -1 < x < 1 \\ -1 & x > 1 \end{cases}$$

Since $f'_+(-1) = -8$ but $f'_-(-1) = 1 \rightarrow f'(-1) \nexists$

$\therefore f$ not diff at $x = -1$

Since $f'_+(1) = -1$ but $f'_-(1) = 8 \rightarrow f'(1) \nexists$

$\therefore f$ not diff at $x = 1$.

11. Given $g(x) = \sqrt{\dfrac{x-1}{4-x}}$, find $g'(x)$.

$$g'(x) = \frac{\dfrac{(4-x)(1)-(x-1)(-1)}{(4-x)^2}}{2\sqrt{\dfrac{x-1}{4-x}}}$$

12. Suppose $h(x) = g(f(x))$ and $f(2) = 6$, $f'(2) = -3$, $g(2) = 4$ and $g'(6) = 8$. Find $h'(2)$.

$$h'(x) = g'(f(x)) \cdot f'(x)$$

$$h'(2) = g'(f(2)) \cdot f'(2)$$

$$= g'(6) \cdot -3$$

$$= (8)(-3)$$

$$= -24$$

Given $f(x) = x^2 + x + 1$, find $f'(x)$ using the definition of derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + x + h + 1 - x^2 - x - 1}{h}$$

$$= \lim_{h \to 0} (2x + h + 1)$$

$$= 2x + 1$$

If f is diff $\to$ f is cont A

If f is not cont $\to$ f not diff A

If f is cont $\to$ f is diff S

Is $f(x) = \sqrt{x}$ diff at $x=0$?

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(0) \nexists \therefore f$ not diff.