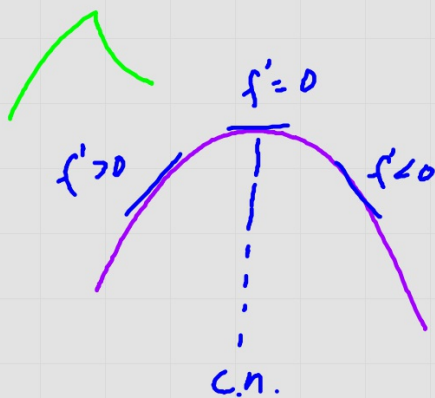
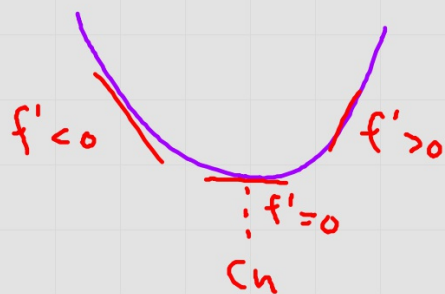


The First Derivative Test for Relative Extrema



If $x=c$ is a c.n. and $f' > 0$ before c and $f' < 0$ after c then $f(c)$ is rel max.



If $x=c$ is a c.n. and $f' < 0$ before c and $f' > 0$ after c then f has a rel min of $f(c)$.

Det where $f(x) = x^3 - 9x^2 + 15x - 5$ is incr/decr.
Find the rel ext.

$$f'(x) = 3x^2 - 18x + 15$$

$$f' \geq 0 \forall x$$

$$f'(x) = 0 \rightarrow x = 1 \text{ and } x = 5$$



Incr/Decr

f is incr on $(-\infty, 1) \cup (5, \infty)$ because
 $f'(x) > 0$ on $(-\infty, 1) \cup (5, \infty)$.

f is decr on $(1, 5)$ because $f'(x) < 0$ on $(1, 5)$.

Rel Ext

f has a rel max of 2 at $x = 1$ because $f'(x) > 0$ on $(-\infty, 1)$
and $f'(x) < 0$ on $(1, 5)$

f has rel min of -30 at $x = 5$ because $f'(x) < 0$ on $(1, 5)$ and
 $f'(x) > 0$ on $(5, \infty)$

$$f(x) = \frac{x-2}{x+2}$$

$$f'(x) = \frac{4}{(x+2)^2}$$

$f'(x) \neq 0$
 $f' \nexists$ at $x = -2$ ($-2 \notin f \therefore$ not c.n.)

Incr/Decr

f is incr on $(-\infty, -2) \cup (-2, \infty)$

because $f'(x) > 0$ on $(-\infty, -2) \cup (-2, \infty)$.

RelExt

f has no rel ext because f has no c.n.

Find rel ext of $f(x) = \sin 2x$ on $[0, \pi]$

$$f'(x) = 2 \cos 2x$$

$$f' \exists \forall x \in [0, \pi]$$

$$f'(x) = 0 \rightarrow x = \frac{\pi}{4} \text{ or } x = \frac{3\pi}{4}$$

f has rel max of 1 at $x = \frac{\pi}{4}$ because
 $f'(x) > 0$ on $(0, \frac{\pi}{4})$ and $f'(x) < 0$ on $(\frac{\pi}{4}, \frac{3\pi}{4})$.

f has rel min of -1 at $x = \frac{3\pi}{4}$ because
 $f'(x) < 0$ on $(\frac{\pi}{4}, \frac{3\pi}{4})$ and $f'(x) > 0$ on $(\frac{3\pi}{4}, \pi)$.

Find rel ext of f $f(x) = \begin{cases} 4 - (x+5)^2 & x < -4 \\ 12 - (x+1)^2 & x \geq -4 \end{cases}$

$$f'(x) = \begin{cases} -2(x+5) & x < -4 \\ -2(x+1) & x \geq -4 \end{cases}$$

-2
6

$$f'(x) = 0 \rightarrow x = -5 \text{ or } x = -1$$

f' $\nexists$ at $x = -4$ because $f'_+(-4) = 6$ but $f'_-(-4) = -2$.

f has rel max of 4 at $x = -5$ because

$f'(x) > 0$ on $(-\infty, -5)$ and $f'(x) < 0$ on $(-5, -4)$.

f has rel min of 3 at $x = -4$ because

$f'(x) < 0$ on $(-5, -4)$ and $f'(x) > 0$ on $(-4, -1)$.

f has rel max of 12 at $x = -1$ because

$f'(x) > 0$ on $(-4, -1)$ and $f'(x) < 0$ on $(-1, \infty)$.