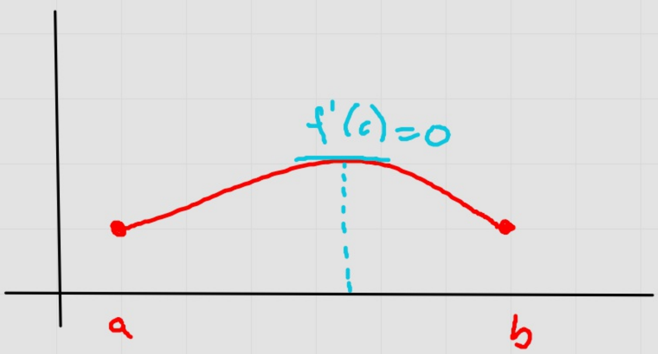
The Mean Value Theorem

Rolle's Thm

If f is cont $[a,b]$ and diff on (a,b)
and if $f(a)=f(b)$ then $\exists\, c \in (a,b)$
such that $f'(c)=0$.



Given $f(x) = x^3 + x^2 - 2x + 1$ on $[-2,0]$. Verify RT holds and find c that satisfies the cond. of R.T.

f is cont on $[-2,0]$ and diff on $(-2,0)$ and $f(-2) = f(0) = 1 \therefore$ RT holds.

$$f'(x) = 3x^2 + 2x - 2$$
$$f'(x) = 0 \rightarrow x = -1.215 \text{ or } x = .549$$

Since $.549 \notin (-2,0)$, $c = -1.215$ only.

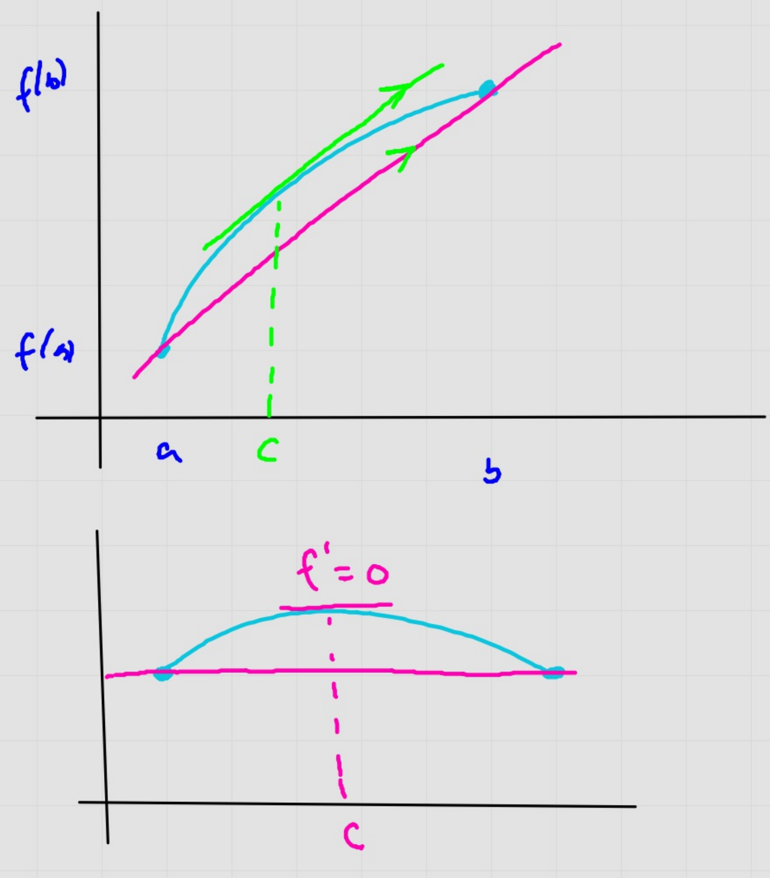
Mean Value Thm

If f is cont on $[a,b]$ and diff on (a,b)
then $\exists\, c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Inst rate of change in fun val at $x=c$

avg rate of change in fun. val. on $[a,b]$


f(b)
f(a)
a
c
b
f' = 0
c

Given $f(x) = 2x^3 + x^2 - x - 1$ on $[0,2]$.

f is cont on $[0,2]$ and diff on $(0,2)$
$\therefore$ MVT holds.

$f'(x) = 6x^2 + 2x - 1$

$$6x^2 - 2x - 1 = \frac{f(2) - f(0)}{2 - 0}$$

$$6x^2 - 2x - 1 = 9 \longrightarrow x = -1.468 \text{ or } x = 1.135$$

Since $-1.468 \notin (0,2)$, $c = 1.135$ only.