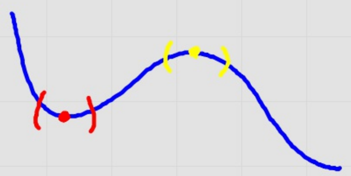
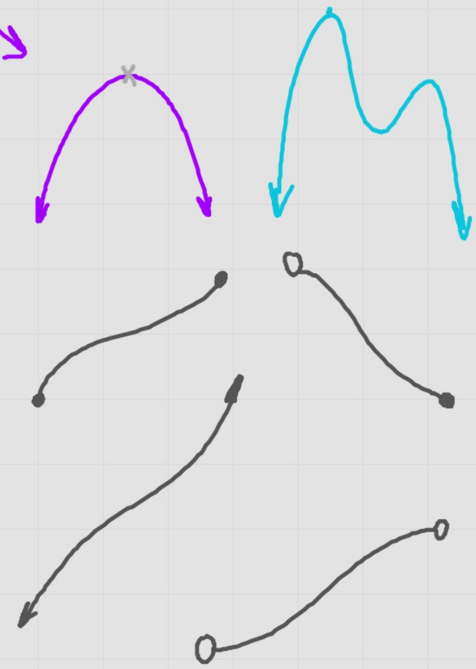
Extrema → function values

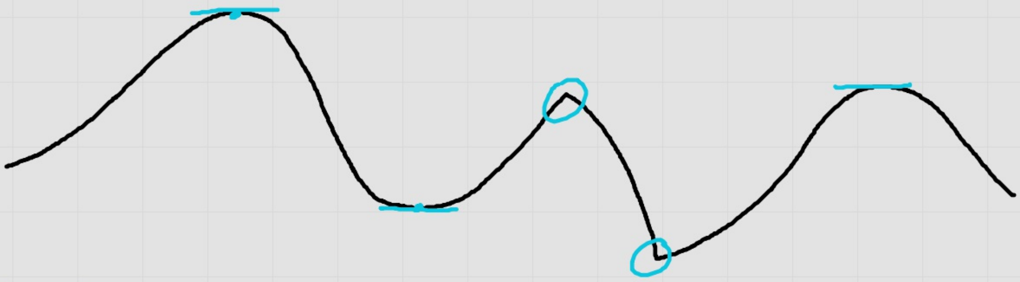
Relative and Absolute ~~Maximums~~ Extrema

local



must occur
on an interval
(never at endpt)





Critical numbers

x-value (in domain of f)
where $f' \nexists$ or $f'(x) = 0$
are called "critical numbers"

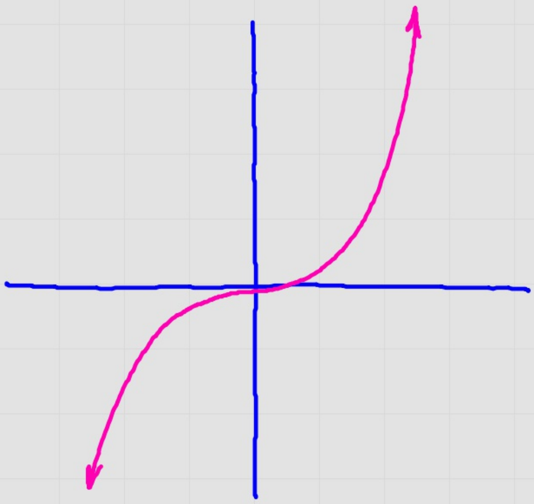
$f(x) = x^3$

$f'(x) = 3x^2$

$f' \exists \forall x$

$f'(x) = 0 \rightarrow x = 0$

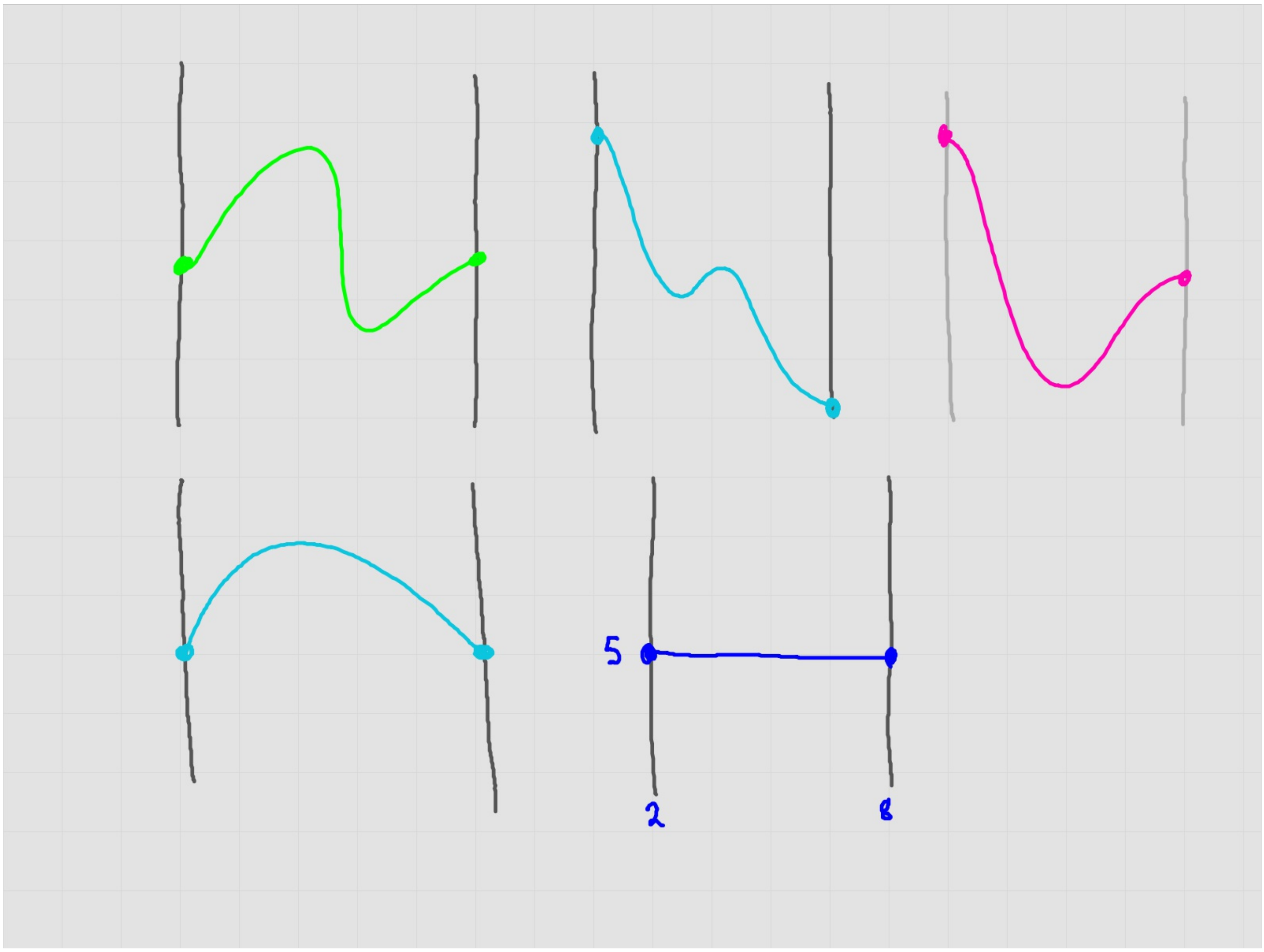
$x = 0$ is a c.n.



If f has rel ext - then they must occur at c.n.

BUT

just because f has c.n. does not necessarily mean f has rel. ext.


5
2
8

Extreme Value Thm

If f is cont. on $[a,b]$
then f must have both
an abs max & an abs min func. value.

To find abs. ext on $[a,b]$

1. f' & c.n.
2. $f(\text{end})$
   $f(\text{c.n})$

Find abs ext of $f(x) = 4x^3 - 15x^2 + 12x$ on $[0,3]$.

$f'(x) = 12x^2 - 30x + 12$

$f \ \exists \ \forall x$

$f'(x) = 0 \rightarrow x = \frac{1}{2}$ or $x = 2$

$f(0) = 0$

$f(3) = 9$

$f(\frac{1}{2}) = \frac{11}{4}$

$f(2) = -4$

$\therefore$ By EVT the abs max is 9 at $x=3$ and abs min is -4 at $x=2$.

Find c.n. of $f(x) = \begin{cases} 3x-6 & x \overset{\geq}{\leq} 3 \\ x^2 & x \overset{\leq}{>} 3 \end{cases}$

$$f'(x) = \begin{cases} 3 & x \overset{>}{<} 3 \\ 2x & x \overset{<}{>} 3 \end{cases}$$

$f'(x) \neq 0$

f' $\nexists$ at $x=3$ because $f'_+(3) = 6$ but $f'_-(3) = 3$.

$\therefore$ $x=3$ is only c.n.