

Derivatives of the Inverse Trigonometric Functions

$$y = \sin^{-1} x$$

$$x = \sin y$$

$$1 = \cos y \quad \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\begin{aligned}\cos y &= \sqrt{1 - \sin^2 y} \\ &= \sqrt{1 - x^2}\end{aligned}$$

$$\boxed{D_x [\sin^{-1} f(x)] = \frac{f'(x)}{\sqrt{1 - f(x)^2}}}$$

$$f(x) = \sin^{-1} e^x$$

$$f'(x) = \frac{e^x}{\sqrt{1 - e^{2x}}}$$

$$y = \cos^{-1} x$$

$$x = \cos y$$

$$1 = -\sin y \frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{1}{\sin y}$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\sin y = \sqrt{1-x^2}$$

$$D_x [\cos^{-1} f(x)] = -\frac{f'(x)}{\sqrt{1-f(x)^2}}$$

$$y = \tan^{-1} x$$

$$x = \tan y$$

$$\begin{aligned} 1 &= \sec^2 y \quad \frac{dy}{dx} \\ \frac{dy}{dx} &= \frac{1}{\sec^2 y} \\ &= \frac{1}{1+x^2} \end{aligned}$$

$$1 + \tan^2 y = \sec^2 y$$

$$1 + x^2 = \sec^2 y$$

$$\boxed{D_x [\tan^{-1} f(x)] = \frac{f'(x)}{1+f(x)^2}}$$

$$\boxed{D_x [\cot^{-1} f(x)] = -\frac{f'(x)}{1+f(x)^2}}$$

$$y = \sec^{-1} x$$

$$x = \sec y$$

$$1 = \sec y \tan y \frac{dy}{dx}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{\sec y \tan y} \\ &= \frac{1}{x \sqrt{x^2 - 1}}\end{aligned}$$

$$1 + \tan^2 y = \sec^2 y$$

$$\begin{aligned}\tan y &= \sqrt{\sec^2 y - 1} \\ &= \sqrt{x^2 - 1}\end{aligned}$$

$$D_x [\sec f(x)] = \frac{f'(x)}{f(x) \sqrt{f(x)^2 - 1}}$$

$$D_x [\csc^{-1} f(x)] = -\frac{f'(x)}{f(x) \sqrt{f(x)^2 - 1}}$$

$$f(x) = \sin^{-1} \sqrt{x}$$

$$\begin{aligned}f'(x) &= \frac{\frac{1}{2\sqrt{x}}}{\sqrt{1-x}} \\&= \frac{1}{2\sqrt{x}\sqrt{1-x}} \\&= \frac{1}{2\sqrt{x-x^2}}\end{aligned}$$

$$h(x) = e^{2x} \tan^{-1} e^{2x}$$

$$h'(x) = (e^{2x}) \frac{2e^{2x}}{1+e^{4x}} + (\tan^{-1} e^{2x})(2e^{2x})$$

$$= \frac{2e^{4x}}{1+e^{4x}} + 2e^{2x} \tan^{-1} e^{2x}$$

$$f(x) = \sin^{-1}(\ln x)$$

$$f'(x) = \frac{\frac{1}{x}}{\sqrt{1 - \ln^2 x}}$$

$$h(x) = \sec^{-1}\left(\underline{\ln e^x}\right) = x$$

$$h'(x) = \frac{1}{x\sqrt{x^2 - 1}}$$