Derivatives of the Inverse Trigonometric Functions

$y = \sin^{-1} x$

$x = \sin y$

$1 = \cos y \dfrac{dy}{dx}$

$\dfrac{dy}{dx} = \dfrac{1}{\cos y}$

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

$\sin^2 y + \cos^2 y = 1$

$\cos y = \sqrt{1-\sin^2 y}$

$= \sqrt{1-x^2}$

$$\boxed{D_x\left[\sin^{-1} f(x)\right] = \frac{f'(x)}{\sqrt{1-f(x)^2}}}$$

$f(x) = \sin^{-1} e^x$

$f'(x) = \dfrac{e^x}{\sqrt{1-e^{2x}}}$

$$y = \cos^{-1} x$$

$$x = \cos y$$

$$1 = -\sin y \frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{1}{\sin y}$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\sin y = \sqrt{1-x^2}$$

$$\boxed{D_x\left[\cos^{-1} f(x)\right] = -\frac{f'(x)}{\sqrt{1-f(x)^2}}}$$

$$y = \tan^{-1} x$$

$$x = \tan y$$

$$1 = \sec^2 y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$= \frac{1}{1+x^2}$$

$$1 + \tan^2 y = \sec^2 y$$

$$1 + x^2 = \sec^2 y$$

$$\boxed{D_x\left[\tan^{-1} f(x)\right] = \frac{f'(x)}{1+f(x)^2}}$$

$$\boxed{D_x\left[\cot^{-1} f(x)\right] = -\frac{f'(x)}{1+f(x)^2}}$$

$$y = \sec^{-1} x$$

$$x = \sec y$$

$$1 = \sec y \tan y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec y \tan y}$$

$$= \frac{1}{x\sqrt{x^2-1}}$$

$$1 + \tan^2 y = \sec^2 y$$

$$\tan y = \sqrt{\sec^2 y - 1}$$

$$= \sqrt{x^2 - 1}$$

$$\boxed{D_x\left[\sec^{-1} f(x)\right] = \frac{f'(x)}{f(x)\sqrt{f(x)^2-1}}}$$

$$\boxed{D_x\left[\csc^{-1} f(x)\right] = -\frac{f'(x)}{f(x)\sqrt{f(x)^2-1}}}$$

$$f(x) = \sin^{-1}\sqrt{x}$$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}}{\sqrt{1-x}}$$

$$= \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$= \frac{1}{2\sqrt{x-x^2}}$$

$$h(x) = e^{2x} \tan^{-1} e^{2x}$$

$$h'(x) = \left(e^{2x}\right) \frac{2e^{2x}}{1+e^{4x}} + \left(\tan^{-1} e^{2x}\right)\left(2e^{2x}\right)$$

$$= \frac{2e^{4x}}{1+e^{4x}} + 2e^{2x} \tan^{-1} e^{2x}$$

$$f(x) = \sin^{-1}(\ln x)$$

$$f'(x) = \frac{\frac{1}{x}}{\sqrt{1 - \ln^2 x}}$$

$$h(x) = \sec^{-1}(\ln e^x) \quad = x$$

$$h'(x) = \frac{1}{x\sqrt{x^2 - 1}}$$