Derivative of Logarithmic Functions

(and the General Exponential Function)

$y = \log_a x \iff a^y = x$

$\log_a a^x = x$

$a^{\log_a x} = x$

$y = \ln x \iff e^y = x$

$\ln e^x = x$

$e^{\ln x} = x$

$\ln 1 = 0$

$\ln e = 1$

$$\log_a xy = \log_a x + \log_a y \qquad \ln xy = \ln x + \ln y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y \qquad \ln \frac{x}{y} = \ln x - \ln y$$

$$\log_a x^n = n \log_a x \qquad \ln x^n = n \ln x$$

$$\log_a x = \frac{\ln x}{\ln a}$$ "change of base"

$$y = \ln x$$

$$e^{y} = x$$

$$e^{y}\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^{y}}$$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$D_x\left[\ln x\right] = \frac{1}{x}$$

$$\boxed{D_x\left[\ln f(x)\right] = \frac{f'(x)}{f(x)}}$$

$$f(x) = \ln(8x+2)$$

$$f'(x) = \frac{8}{8x+2}$$

$$f(x) = \ln \frac{5x-1}{3x+2}$$

$$f(x) = \ln(5x-1) - \ln(3x+2)$$

$$f'(x) = \frac{5}{5x-1} - \frac{3}{3x+2}$$

$$\frac{\dfrac{(\ x\ )-(\ x\ )}{(\ \ )^2}}{\dfrac{(\ \ )}{(\ \ )}}$$

$$g(x) = \ln \sqrt[3]{\sec x}$$

$$= \frac{1}{3} \ln(\sec x)$$

$$g'(x) = \frac{1}{3} \frac{\sec x \tan x}{\sec x}$$

$$= \frac{1}{3} \tan x$$

$$y = \log_a x = \frac{\ln x}{\ln a}$$

$$\frac{x^3}{7}$$

$$y = \frac{\ln x}{\ln a}$$

$$D_x[\ln x] = \frac{1}{x}$$

$$y = \frac{1}{\ln a}\ln x$$

$$D_x[\log_a x] = \frac{1}{x \ln a}$$

$$\frac{dy}{dx} = \frac{1}{x \ln a}$$

$$D_x[\log_a f(x)] = \frac{f'(x)}{f(x)\ln a} = \frac{f'(x)\log_a e}{f(x)}$$

$$f(x) = \log_3(2x-1)$$

$$f'(x) = \frac{2}{(2x-1)\ln 3} = \frac{2\log_3 e}{2x-1}$$

$$\log_a x = \frac{\ln x}{\ln a}$$

For $x = e$

$$\log_a e = \frac{\ln e}{\ln a}$$

$$\log_a e = \frac{1}{\ln a}$$

$$y = a^x$$

$$a^x = e^{\ln a^x}$$

$$a^x = e^{x \ln a}$$

$$D_x[a^x] = D_x[e^{x \ln a}]$$

$$= e^{x \ln a} \ln a$$

$$= e^{\ln a^x} \ln a$$

$$= a^x \ln a$$

$$D_x[e^x] = e^x$$

$$D_x[a^x] = a^x \ln a$$

$$\boxed{D_x[a^{f(x)}] = a^{f(x)} f'(x) \ln a}$$

$$f(x) = 8^{x^2+5x}$$

$$f'(x) = \left(8^{x^2+5x}\right)(2x+5)(\ln 8)$$

$$g(x) = 7^{\sin x}$$

$$g'(x) = \left(7^{\sin x}\right)(\cos x)(\ln 7)$$

$f(x) = x^8$ fun$^{\#}$

$g(x) = 8^x$ $\#^{\text{func.}}$

$y = x^{x^3}$ "logarithmic differentiation"

$\ln y = x^3 \ln x$

$\frac{1}{y}\frac{dy}{dx} = \frac{x^3}{x} + 3x^2 \ln x$

$\frac{dy}{dx} = y(x^2 + 3x^2 \ln x)$

$\frac{dy}{dx} = x^{x^3}(x^2 + 3x^2 \ln x)$

$$y = \frac{3x-2}{5x+2}$$

$\ln x^3 + \ln(\sin x)$

$3\ln x$

$$\ln y = \ln(3x-2) - \ln(5x+2)$$

$$\frac{dy}{dx} = \left(\frac{3x-2}{5x+2}\right)\left(\frac{3}{3x-2} - \frac{5}{5x+2}\right)$$

$$y = x^3 \sin x$$

$$\frac{dy}{dx} = (x^3 \sin x)\left(\frac{3}{x} + \cot x\right)$$

$$3^{5^x} = 6$$

$$\ln 3^{5^x} = \ln 6$$

$$5^x \ln 3 = \ln 6$$

$$5^x = \frac{\ln 6}{\ln 3}$$

$$\ln 5^x = \ln\left(\frac{\ln 6}{\ln 3}\right)$$

$$x = \frac{\ln\left(\frac{\ln 6}{\ln 3}\right)}{\ln 5}$$

$$\log_3 3^{5^x} = \log_3 6$$

$$5^x = \log_3 6$$

$$\log_5 5^x = \log_5(\log_3 6)$$

$$x = \log_5(\log_3 6)$$