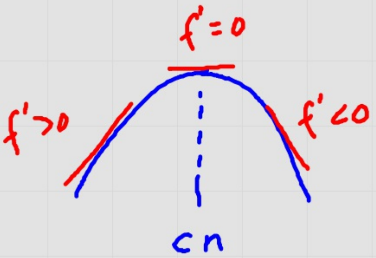
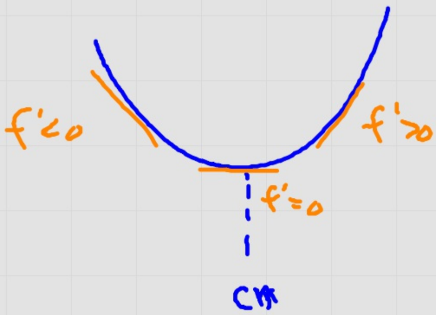
The First Derivative Test for Relative Extrema



If $x=c$ is a c.n and $f'>0$ before c and $f'<0$ after c then f has a rel max $f(c)$.



If $x=c$ is a cn and $f'<0$ before c and $f'>0$ after c f has a rel min of $f(c)$.

Det where $f(x)=x^3-9x^2+15x-5$ is incr/decr.
and find any rel. ext.

$$f'(x)=3x^2-18x+15$$

$$f' \exists \ \forall x$$

$$f'(x)=0 \rightarrow x=1 \text{ a } x=5$$

Incr/Decr

f is incr on $(-\infty,1)\cup(5,\infty)$ because $f'(x)>0$ on $(-\infty,1)\cup(5,\infty)$.

f is decr on $(1,5)$ because $f'(x)<0$ on $(1,5)$.

Rel Ext

f has a rel max of 2 at $x=1$ because
$f'(x)>0$ on $(-\infty,1)$ and $f'(x)<0$ on $(1,5)$

f has a rel min of -30 at $x=5$ because $f'(x)<0$ on $(1,5)$
and $f'(x)>0$ on $(5,\infty)$.

Det. where $f(x) = \frac{x-2}{x+2}$ is incr/decr. and find any rel ext.

$$f'(x) = \frac{4}{(x+2)^2}$$

$f' \nexists$ at $x=-2$ $\left(-2 \notin f \therefore \text{not c.n.}\right)$

$f'(x) \neq 0$

<u>Rel Ext</u>

f has no rel. ext because f has no c.n.

<u>Incr / Decr</u>

f is incr. on $(-\infty,-2) \cup (-2,\infty)$ because

$f'(x) > 0$ on $(-\infty,-2) \cup (-2,\infty)$.

Find the rel. ext of $f(x) = \sin 2x$ on $[0, \pi]$.

$f'(x) = 2\cos 2x$

$f' \exists\ \forall x \in [0, \pi]$

$f'(x) = 0 \rightarrow x = \frac{\pi}{4}$ or $x = \frac{3\pi}{4}$

f has rel max of 1 at $x = \frac{\pi}{4}$ because

$f'(x) > 0$ on $(0, \frac{\pi}{4})$ and $f'(x) < 0$ on $(\frac{\pi}{4}, \frac{3\pi}{4})$.

f has a rel min of -1 $x = \frac{3\pi}{4}$ because

$f'(x) < 0$ on $(\frac{\pi}{4}, \frac{3\pi}{4})$ and $f'(x) > 0$ on $(\frac{3\pi}{4}, \pi)$.

Find rel ext of $f(x) = \begin{cases} 4-(x+5)^2 & x < -4 \\ 12-(x+1)^2 & x \geq -4 \end{cases}$

-2
6

$f'(x) = \begin{cases} -2(x+5) & x < -4 \\ -2(x+1) & x > -4 \end{cases}$

$f'(x) = 0 \rightarrow x = -5$ or $x = -1$

$f' \nexists$ at $x = -4$ because $f'_+(-4) = 6$ but $f'_-(-4) = -2$.

f has rel max of at $x = -5$ because
$f'(x) > 0$ on $(-\infty, -5)$ and $f'(x) < 0$ on $(-5, -4)$.

f has a rel min of at $x = -4$ because
$f'(x) < 0$ on $(-5, -4)$ and $f'(x) > 0$ on $(-4, -1)$.

f has a rel max of at $x = -1$ because
$f'(x) > 0$ on $(-4, -1)$ and $f'(x) < 0$ on $(-1, \infty)$.