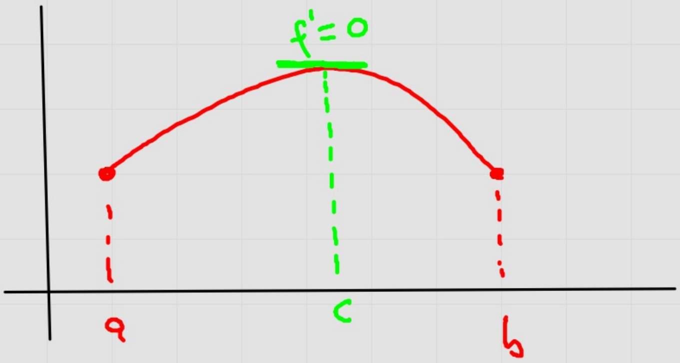
The Mean Value Theorem

Rolle's Thm

If f is cont $[a,b]$ and diff on (a,b)
and if $f(a)=f(b)$ then $\exists\, c\in(a,b)$
such that $f'(c)=0$.



Given $f(x) = x^3 + x^2 - 2x + 1$ on $[-2,0]$. Verify that R.T holds and find the value of c that satisfies the concl. of R.T.

f is cont on $[-2,0]$ and diff. on $(-2,0)$ and $f(-2) = f(0) = 1 \therefore$ RT holds.

$f'(x) = 3x^2 + 2x - 2$

$f'(x) = 0 \to x = -1.215$ or $x = .549$

Since $.549 \notin (-2,0)$, $c = -1.215$ only.

Mean Value Thm

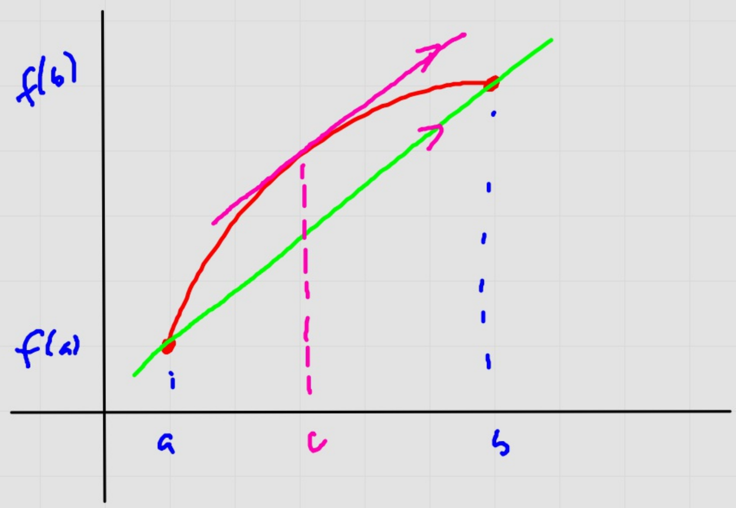
If f is cont $[a,b]$ and diff on (a,b)

then $\exists\ c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

inst. rate of change in fun. val at $x=c$.

avg rate of change in fun. val. on $[a,b]$



f(b)
f(a)
a
c
b

Given $f(x) = 2x^3 + x^2 - x - 1$ on $[0,2]$.

f is cont on $[0,2]$ and f is diff on $(0,2)$.

$$f'(x) = 6x^2 + 2x - 1$$

$$6x^2 + 2x - 1 = \frac{f(2) - f(0)}{2 - 0}$$

$$6x^2 + 2x - 1 = 9 \rightarrow x = -1.468 \text{ or } x = 1.135$$

Since $-1.468 \notin (0,2)$, $c = 1.135$ only.