Derivative of Logarithmic Functions

(and the General Exponential Function)

$y = \log_a x \iff a^y = x$

$\log_a a^x = x$

$a^{\log_a x} = x$

$y = \ln x \iff e^y = x$

$\ln e^x = x$

$e^{\ln x} = x$

$\ln 1 = 0$

$\ln e = 1$

$$\log_a AB = \log_a A + \log_a B$$

$$\log_a \frac{A}{B} = \log_a A - \log_a B$$

$$\log_a B^n = n \log_a B$$

$$\boxed{\log_a x = \frac{\ln x}{\ln a}}$$ change of base formula

$$2^{3^x} = 5$$

$$\ln 2^{3^x} = \ln 5$$

$$3^x \ln 2 = \ln 5$$

$$3^x = \frac{\ln 5}{\ln 2}$$

$$\ln 3^x = \frac{\ln 5}{\ln 2}$$

$$x \ln 3 = \frac{\ln 5}{\ln 2}$$

$$x = \frac{\frac{\ln 5}{\ln 2}}{\ln 3}$$

$$\log_2 2^{3^x} = \log_2 5$$

$$3^x = \log_2 5$$

$$\log_3 3^x = \log_3(\log_2 5)$$

$$x = \log_3(\log_2 5)$$

$$y = \ln x$$

$$e^y = x$$

$$e^y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y}$$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$D_x[\ln x] = \frac{1}{x}$$

$$\boxed{D_x[\ln f(x)] = \frac{f'(x)}{f(x)}}$$

$$f(x) = \ln(5x-1)$$

$$f'(x) = \frac{5}{5x-1}$$

$$f(x) = \ln \sqrt[3]{\frac{3x-1}{5x-7}}$$

$$= \frac{1}{3}\left[\ln(3x-1) - \ln(5x-7)\right]$$

$$f'(x) = \frac{1}{3}\left[\frac{3}{3x-1} - \frac{5}{5x-7}\right]$$

$$\frac{\frac{1}{3}\left(\frac{3x-1}{5x-7}\right)\left(\frac{(\quad x\quad)-(\quad x)}{(\quad)^2}\right)}{\sqrt[3]{\frac{(\quad)}{(\quad)}}}$$

$y = \log_a x$

$D_x[\log_a x] = \dfrac{1}{x \ln a}$

$\log_a x = \dfrac{\ln x}{\ln a}$

$D_x[\ln x] = \dfrac{1}{x}$

$$D_x[\log_a x] = D_x\left[\frac{\ln x}{\ln a}\right]$$

$$= \frac{1}{\ln a} D_x[\ln x]$$

$$= \frac{1}{x \ln a}$$

$$f(x) = \log_3(x^2 - 5x)$$

$$f'(x) = \frac{2x-5}{(x^2-5x)\ln 3} = \frac{(2x-5)\log_3 e}{x^2-5x}$$

$$\log_a x = \frac{\ln x}{\ln a}$$

For $x = e$

$$\log_a e = \frac{\ln e}{\ln a}$$

$$\log_a e = \frac{1}{\ln a}$$

$$a^x = e^{\ln a^x}$$

$$a^x = e^{x \ln a}$$

$$D_x\left[a^x\right] = D_x\left[e^{x \ln a}\right]$$

$$= e^{x \ln a} \cdot \ln a$$

$$= e^{\ln a^x} \ln a$$

$$= a^x \ln a$$

$$D_x\left[e^x\right] = e^x$$

$$D_x\left[a^x\right] = a^x \ln a$$

$$\boxed{D_x\left[a^{f(x)}\right] = a^{f(x)} f'(x) \ln a}$$

$$D_x\left[e^{f(x)}\right] = e^{f(x)} f'(x)$$

$$D_x\left[a^{f(x)}\right] = a^{f(x)} f'(x) \ln a$$

$$D_x\left[\ln f(x)\right] = \frac{f'(x)}{f(x)}$$

$$D_x\left[\log_a f(x)\right] = \frac{f'(x)}{f(x) \ln a} = \frac{f'(x) \log_a e}{f(x)}$$

$$f(x) = 8^{x^3}$$

$$f'(x) = \left(8^{x^3}\right)\left(3x^2\right)\left(\ln 8\right)$$

$$y = x^{\sin x}$$

"logarithmic differentiation"

$$\ln y = \sin x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\sin x}{x} + (\ln x)\cos x$$

$$\frac{dy}{dx} = y\left[\frac{\sin x}{x} + (\ln x)\cos x\right]$$

$$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + (\ln x)\cos x\right].$$

$$y = \sqrt[3]{\frac{5x+2}{7x-1}}$$

$$\rightarrow \ln y = \frac{1}{3}\left[\ln(5x+2) - \ln(7x-1)\right]$$

$$\rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{3}\left[\frac{5}{5x+2} - \frac{7}{7x-1}\right]$$

$$\frac{dy}{dx} = \frac{1}{3}\sqrt[3]{\frac{5x+2}{7x-1}}\left[\frac{5}{5x+2} - \frac{7}{7x-1}\right]$$

$$y = \underbrace{(5x+2)}^{10}$$

10 ln

$$\frac{dy}{dx} = 10\left[\frac{5}{5x+2}\right]$$