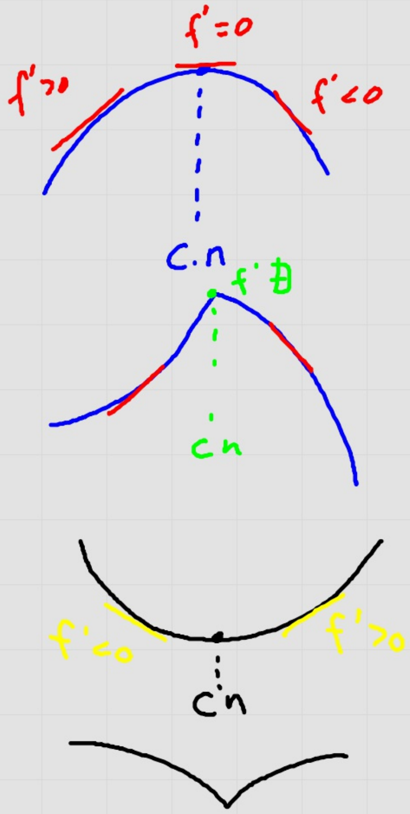
The First Derivative Test for Relative Extrema

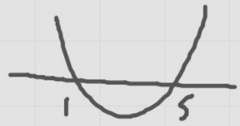


If $x=c$ is c.n. and
$f'(x)>0$ before c and
$f'(x)<0$ after c then
f has a rel max of $f(c)$.

If $x=c$ is a cn and
$f'(x)<0$ before c and
$f'(x)>0$ after c then
f has a rel min of $f(c)$.

Det. where $f(x) = x^3 - 9x^2 + 15x - 5$ is incr/decr.

$f(x) = 3x^2 - 18x + 15$



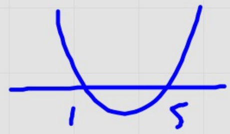
$f' \exists \ \forall x$

$f'(x) = 0 \Rightarrow x = 1$ or $x = 5$

f is incr on $(-\infty, 1) \cup (5, \infty)$ because $f'(x) > 0$ on $(-\infty, 1) \cup (5, \infty)$.

f is decr on $(1, 5)$ because $f'(x) < 0$ on $(1, 5)$.

Same function – find the rel. ext.



$f'(x) = 3x^2 - 18x + 15$

$f' \exists \ \forall x$

$f'(x) = 0 \Rightarrow x = 1$ a $x = 5$

f has a rel max of 2 at $x=1$
because $f'(x) > 0$ on $(-\infty, 1)$ and $f'(x) <$ on $(1,5)$.

f has a rel min of -30 at $x=5$
because $f'(x) < 0$ on $(1,5)$ and $f'(x) > 0$ on $(5, \infty)$.

Det. where $f(x)=\dfrac{x-2}{x+2}$ is incr/decr and find any rel. ext.

$$f'(x)=\frac{4}{(x+2)^2}$$

$$f'(x)\neq 0$$

$$f' \nexists \text{ at } x=-2 \quad (-2 \notin f \therefore \text{not c.n.})$$

f is incr on $(-\infty,-2)\cup(-2,\infty)$ because $f'(x)>0$ on $(-\infty,-2)\cup(-2,\infty)$.

f has no rel ext because f has no c.n.

Find rel ext of $f(x) = \sin 2x$ on $[0, \pi]$.

$f'(x) = 2\cos 2x$

$f' \exists \ \forall x \in [0, \pi]$

$f'(x) = 0 \rightarrow x = \frac{\pi}{4}$ a $x = \frac{3\pi}{4}$

f has a rel max of 1 at $x = \frac{\pi}{4}$
because $f'(x) > 0$ on $(0, \frac{\pi}{4})$ and $f'(x) < 0$ on $(\frac{\pi}{4}, \frac{3\pi}{4})$.

f has a rel min of -1 at $x = \frac{3\pi}{4}$ because
$f'(x) < 0$ on $(\frac{\pi}{4}, \frac{3\pi}{4})$ and $f'(x) > 0$ on $(\frac{3\pi}{4}, \pi)$.

Find rel ext of $f(x) = \begin{cases} 4-(x+5)^2 & x<-4 \\ 12-(x+1)^2 & x \geq -4 \end{cases}$

-2

6

$f'(x) = \begin{cases} -2(x+5) & x<-4 \\ -2(x+1) & x>-4 \end{cases}$

$f'(x) = 0 \Rightarrow x=-5$ or $x=-1$

f' $\nexists$ at $x=4$ because $f'_+(-4)=6$ but $f'_-(-4)=-2$.

f has a rel max of 4 at $x=-5$ because
$f'(x)>0$ on $(-\infty,-5)$ and $f'(x)<0$ on $(-5,-4)$

f has a rel min of 3 at $x=-4$ because
$f'(x)<0$ on $(-5,-4)$ and $f'(x)>0$ on $(-4,-1)$

f has rel max of 12 at $x=-1$ because $f'(x)>0$ on $(-4,-1)$ and $f'(x)<0$ on $(-1,\infty)$