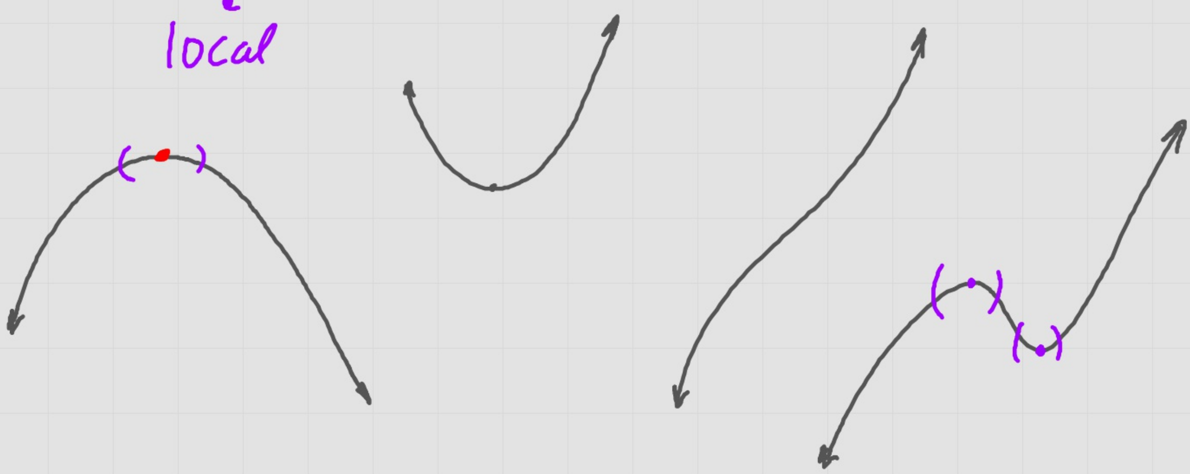
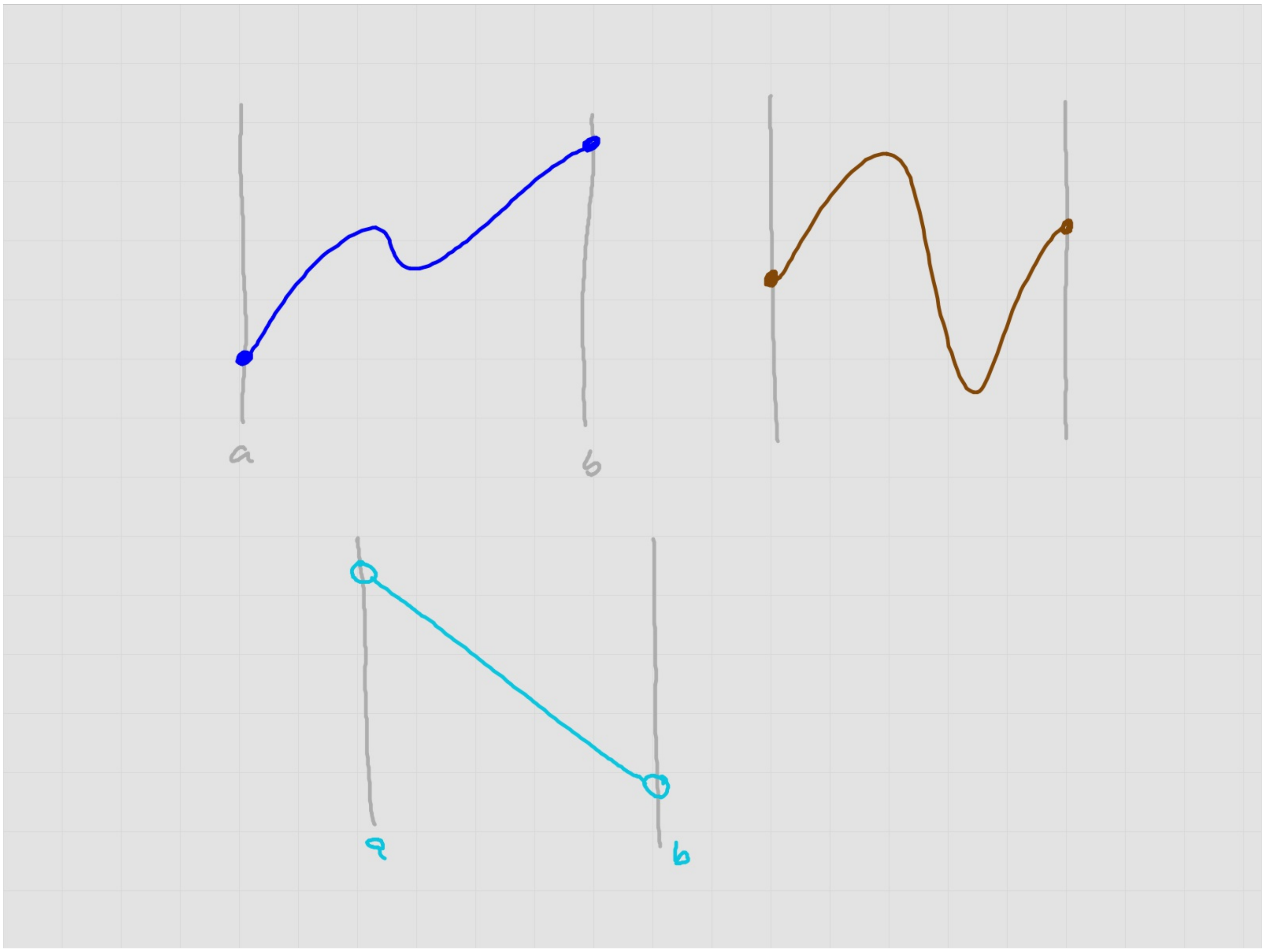
Extrema → FUNCTION VALUES

Relative and Absolute Maximums

local

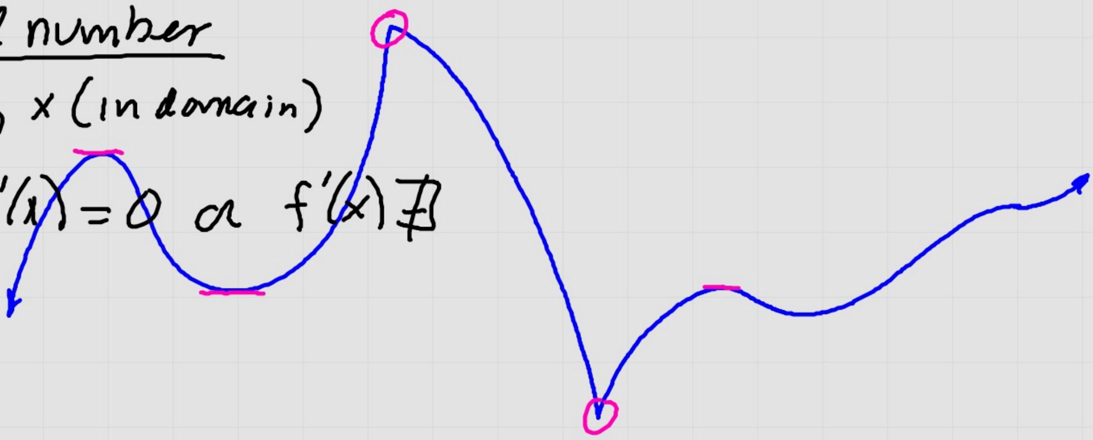



a
b
a
b

Critical number

Value of x (in domain)

where $f'(x) = 0$ or $f'(x) \nexists$



If f has rel. ext they must occur at c.n. BUT

just because f has c.n does not necessarily mean f has rel. ext.

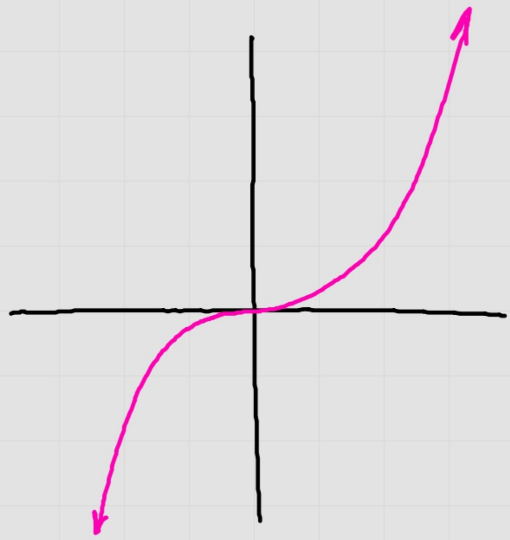
$f(x) = x^3$

$f'(x) = 3x^2$

$f' \exists \forall x$

$f'(x) = 0 \to x = 0$

$x = 0$ is a c.n.



$$f(x) = \frac{3}{x-5}$$

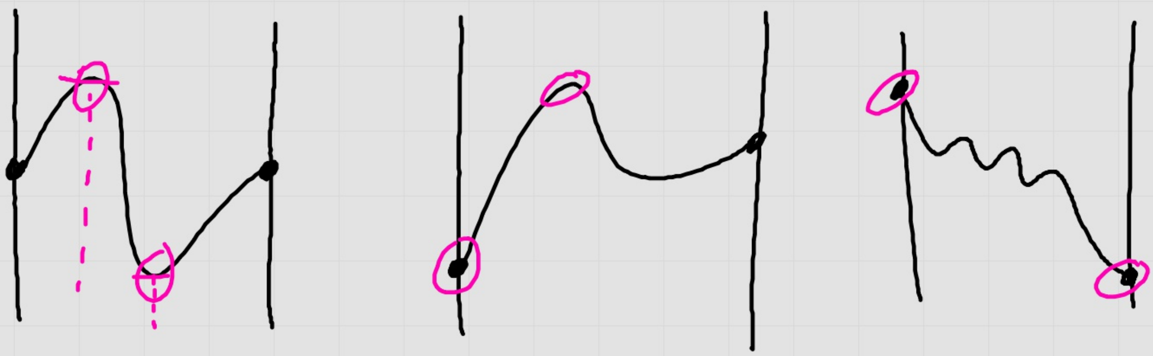
$$f'(x) = -3(x-5)^{-2}(1)$$

$$= -\frac{3}{(x-5)^2}$$

$f'(x) \neq 0$

$f'(x) \nexists$ at $x=5$

But $5 \notin f$ $\therefore$ $x=5$ not c.n.



EXTREME VALUE THM

If f is cont. on $[a,b]$ then f has both an abs. max func. val **AND** an abs. min func. val.

Find the abs. ext of $f(x) = 4x^3 - 15x^2 + 12x$ on $[0,3]$.

$f'(x) = 12x^2 - 30x + 12$

$f' \exists \forall x$

$f'(x) = 0 \longrightarrow x = \frac{1}{2}$ or $x = 2$

$f(0) = 0$

$f(3) = 9$

$f(\frac{1}{2}) = \frac{11}{4}$

$f(2) = -4$

$\therefore$ By EVT abs max is 9 at $x=3$

and abs min is -4 at $x=2$.

To find abs ext on [a,b].

1. Find f' and c.n. (in interval)

2. f(end pts) & f(c.n.)

3. Big fun. val is abs max
Smallest fun val is abs min.

Find c.n of $f(x) = \begin{cases} 3x-6 & x \geq 3 \\ x^2 & x < 3 \end{cases}$ $\begin{matrix} \leq \\ > \end{matrix}$

$f'(x) = \begin{cases} 3 & x > 3 \\ 2x & x < 3 \end{cases}$

$f'(x) = 0 \rightarrow x = 0$

$f'(3) \nexists$ because $f'_+(3) = 3$ but $f'_-(3) = 6$.

$\therefore$ $x = 0$ and $x = 3$ are c.n.