Exponential Growth

$$\frac{dy}{dt} = ky$$

$$\frac{1}{y}\frac{dy}{dt} = k$$

$$\ln|y| = kt + C$$

$$y = e^{kt+C}$$

$$y = e^{kt}e^{C}$$

$$y = Ae^{kt}$$

For $t = 0 \rightarrow y = y_0$

$$y_0 = Ae^{0}$$

$$\boxed{y = y_0 e^{kt}}$$

(Ex) $t=0$ $y=1000$ $(0, 1000)$

$t=12$ $y=2000$ $(12, 2000)$

How long before we have 10^6 bac.?

$y = y_0 e^{kt}$

$2000 = 1000 e^{12k}$

$k = \frac{\ln 2}{12}$

$y = 1000 e^{\frac{\ln 2}{12} t}$

For $y = 10^6$

$10^6 = 10^3 e^{\frac{\ln 2}{12} t}$

$10^3 = e^{\frac{\ln 2}{12} t}$

$t = 119.589$

$\therefore 119.589$ min.

How many bac. after 3 hrs?

$$y = 1000e^{\frac{\ln 2}{12}t}$$

For $t = 180$

$$y = 32768000$$

$\therefore$ 32,768,000 Bacteria

1950: 50 000 $\rightarrow$ $(0, 50000)$

1980: 75000 $\rightarrow$ $(30, 75000)$

2016: ?

$y = y_0 e^{kt}$

$75000 = 50000 e^{30k}$

$k = \frac{\ln \frac{3}{2}}{30}$

$y = 50000 e^{\frac{\ln \frac{3}{2}}{30} t}$

For $t = 66$

$y = 122003.074$

Doubling Time

$$2y_0 = y_0 e^{kt}$$

$$2 = e^{kt}$$

$$\ln 2 = kt$$

$$t = \frac{\ln 2}{k} \qquad k = \frac{\ln 2}{t}$$

$$A = P_0 e^{rt}$$

one time invest

$$A = \frac{P_0}{k}\left(e^{rt} - 1\right)$$

yearly invest