Derivatives of the Inverse Trigonometric Functions

$$y = \sin^{-1} x$$

$$x = \sin y$$

$$1 = \cos y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\sin^2 y + \cos^2 y = 1$$

$$x^2 + \cos^2 y = 1$$

$$\cos y = \sqrt{1 - x^2}$$

$$\boxed{D_x\left[\sin^{-1} f(x)\right] = \frac{f'(x)}{\sqrt{1 - f(x)^2}}}$$

$$f(x) = \sin^{-1} e^{2x}$$

$$f'(x) = \frac{2e^{2x}}{\sqrt{1 - e^{4x}}}$$

$y = \cos^{-1} x$

$x = \cos y$

$1 = -\sin y \frac{dy}{dx}$

$\frac{dy}{dx} = -\frac{1}{\sin y}$

$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$

$\sin^2 y + \cos^2 y = 1$

$\sin^2 y + x^2 = 1$

$\sin y = \sqrt{1-x^2}$

$$\boxed{D_x\left[\cos^{-1} f(x)\right] = -\frac{f'(x)}{\sqrt{1-f(x)^2}}}$$

$$y = \tan^{-1} x$$

$$x = \tan y$$

$$1 = \sec^2 y \, \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$1 + \tan^2 y = \sec^2 y$$

$$1 + x^2 = \sec^2 y$$

$$\boxed{\begin{aligned} D_x\left[\tan^{-1} f(x)\right] &= \frac{f'(x)}{1+f(x)^2} \\ D_x\left[\cot^{-1} f(x)\right] &= -\frac{f'(x)}{1+f(x)^2} \end{aligned}}$$

$$y = \sec^{-1} x$$

$$x = \sec y$$

$$1 = \sec y \tan y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec y \tan y}$$

$$\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$$

$$1 + \tan^2 y = \sec^2 y$$

$$\tan y = \sqrt{\sec^2 y - 1}$$

$$= \sqrt{x^2 - 1}$$

$$\boxed{\begin{aligned} D_x\left[\sec^{-1} f(x)\right] &= \frac{f'(x)}{f(x)\sqrt{f(x)^2-1}} \\ D_x\left[\csc^{-1} f(x)\right] &= -\frac{f'(x)}{f(x)\sqrt{f(x)^2-1}} \end{aligned}}$$

$$f(x) = \sin^{-1}\sqrt{x}$$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}}{\sqrt{1-x}}$$

$$= \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$= \frac{1}{2\sqrt{x-x^2}}$$

$$f(x) = \tan^{-1}(3x+5)$$

$$f'(x) = \frac{3}{1+(3x+5)^2}$$

$$g(x) = \cos^{-1} 5x$$

$$g'(x) = -\frac{5}{\sqrt{1-25x^2}}$$

$$h(x) = e^{-x}\sec^{-1}(e^{-x})$$

$$h'(x) = (e^{-x})\frac{-e^{-x}}{e^{-x}\sqrt{e^{-2x}-1}} + \left[\sec^{-1}(e^{-x})\right](-e^{-x})$$

$$f(x) = \ln\left[\tan^{-1} x^2\right]$$

$$f'(x) = \frac{\frac{2x}{1+x^4}}{\tan^{-1} x^2}$$