Derivative of Logarithmic Functions

(and the General Exponential Function)

$y = \log_a x \iff a^y = x$

$\log_a a^x = x$

$a^{\log_a x} = x$

$y = \ln x \iff e^y = x$

$\ln e^x = x$

$e^{\ln x} = x$

$\ln 1 = 0$

$\ln e = 1$

$$\log_a xy = \log_a x + \log_a y$$

$$\ln xy = \ln x + \ln y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\ln \frac{x}{y} = \ln x - \ln y$$

$$\log_a x^n = n \log_a x$$

$$\ln x^n = n \ln x$$

$$\log_a x = \frac{\ln x}{\ln a}$$ "change of base"

$$y = \ln x$$

$$e^{y} = x$$

$$e^{y}\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^{y}}$$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$D_x[\ln x] = \frac{1}{x}$$

$$\boxed{D_x[\ln f(x)] = \frac{f'(x)}{f(x)}}$$

$$f(x) = \ln(5x-3)$$

$$f'(x) = \frac{5}{5x-3}$$

$$g(x) = \ln x^{3}$$

$$= 3\ln x$$

$$g'(x) = \frac{3}{x}$$

$$y = \ln \frac{3x+4}{7x-1}$$

$$y = \ln(3x+4) - \ln(7x-1)$$

$$\frac{dy}{dx} = \frac{3}{3x+4} - \frac{7}{7x-1}$$

$$\frac{\frac{(\ x\ )-(\ x\ )}{(\quad)^2}}{\frac{(\quad)}{(\quad)}}$$

$$D_x[\ln x] = \frac{1}{x}$$

$$y = \log_a x$$

$$\log_a x = \frac{\ln x}{\ln a}$$

$$D_x[\log_a x] = D_x\left[\frac{\ln x}{\ln a}\right]$$

$$= \frac{1}{\ln a}\frac{1}{x}$$

$$f(x) = \log_7(5x-2)$$

$$f'(x) = \frac{5}{(5x-2)\ln 7} = \frac{5\log_7 e}{5x-2}$$

$$D_x[\log_a x] = \frac{1}{x \ln a}$$

$$D_x[\log_a f(x)] = \frac{f'(x)}{f(x)\ln a}$$

$$= \frac{f'(x)\log_a e}{f(x)}$$

$$\log_a x = \frac{\ln x}{\ln a}$$

let $x = e$

$$\log_a e = \frac{\ln e}{\ln a}$$

$$\log_a e = \frac{1}{\ln a}$$

$$y = a^x$$

$$D_x[e^x] = e^x$$

$$D_x[a^x] = a^x \ln a$$

$$a^x = e^{\ln a^x}$$

$$\boxed{D_x\left[a^{f(x)}\right] = a^{f(x)} f'(x) \ln a}$$

$$a^x = e^{x \ln a}$$

$$D_x[a^x] = D_x\left[e^{x \ln a}\right]$$

$$= e^{x \ln a} \ln a$$

$$= e^{\ln a^x} \ln a$$

$$= a^x \ln a$$

$$f(x) = 8^{x^2+5x}$$

$$f'(x) = \left(8^{x^2+5x}\right)(2x+5)(\ln 8)$$

fun$^{\#}$ $(3x-5)^{10}$

$\#^{\text{fun}}$ 8^{x^2}

fun$^{\text{fun}}$

$$y = x^{x^2}$$

$$\ln y = \ln x^{x^2}$$

$$\ln y = x^2 \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{x^2}{x} + 2x \ln x$$

"logarithmic differentiation"

$$\frac{dy}{dx} = y\left[x + 2x \ln x\right]$$

$$\frac{dy}{dx} = x^{x^2}\left[x + 2x \ln x\right]$$

$$y = \sqrt[5]{\frac{5x+2}{9x-1}}$$

$$\ln y = \frac{1}{5}\left[\ln(5x+2) - \ln(9x-1)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{5}\left[\frac{5}{5x+2} - \frac{9}{9x-1}\right]$$

$$\frac{dy}{dx} = \frac{1}{5}\left(\frac{5x+2}{9x-1}\right)\left(\frac{5}{5x+2} - \frac{9}{9x-1}\right)$$

$$3^{2^x} = 4$$

$$\ln 3^{2^x} = \ln 4$$

$$2^x \ln 3 = \ln 4$$

$$2^x = \frac{\ln 4}{\ln 3}$$

$$\ln 2^x = \ln\left(\frac{\ln 4}{\ln 3}\right)$$

$$x \ln 2 = \ln\left(\frac{\ln 4}{\ln 3}\right)$$

$$x = \frac{\ln\left(\frac{\ln 4}{\ln 3}\right)}{\ln 2}$$

$$3^{2^x} = 4$$

$$\log_3 3^{2^x} = \log_3 4$$

$$2^x = \log_3 4$$

$$\log_2 2^x = \log_2(\log_3 4)$$

$$x = \log_2(\log_3 4)$$

$$5^x = 6$$

$$\log_5 5^x = \log_5 6$$

$$x = \log_5 6$$