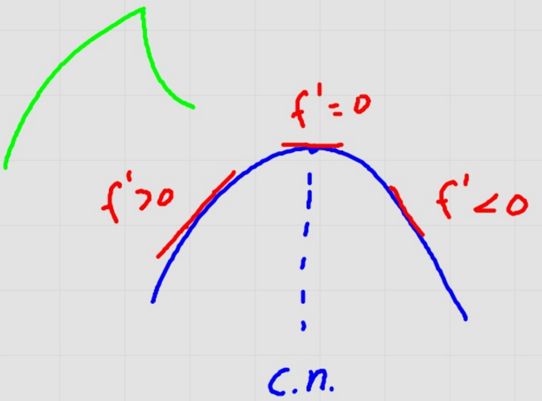
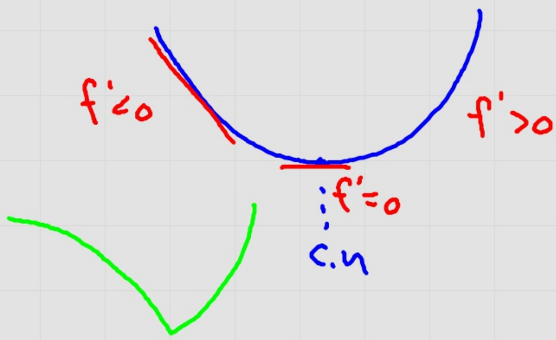
The First Derivative Test for Relative Extrema



If $x=c$ is a c.n and
$f' > 0$ before c and
$f' < 0$ after c then
$f(c)$ is rel max.



If $x=c$ is a c.n and
$f' < 0$ before c and
$f' > 0$ after c then
$f(c)$ is a rel mi

Given $f(x) = x^3 - 9x^2 + 15x - 5$, det. when f is incr/decr.

$f'(x) = 3x^2 - 18x + 15$

$f' \exists \, \forall x$

$f'(x) = 0 \rightarrow x = 1$ or $x = 5$

f is incr on $(-\infty, 1) \cup (5, \infty)$
because $f'(x) > 0$ on $(-\infty, 1) \cup (5, \infty)$.

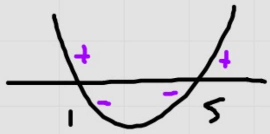
f is decreas. on $(1, 5)$ because $f'(x) < 0$ on $(1, 5)$.

Find the rel. ext of $f(x) = x^3 - 9x^2 + 15x - 5$.

$f'(x) = 3x^2 - 18x + 15$

$f' \; \exists \; \forall x$

$f'(x) = 0 \to x = 1$ a $x = 5$



f has a rel max of 2 at $x = 1$ because $f'(x) > 0$ on $(-\infty, 1)$ and $f'(x) < 0$ on $(1, 5)$.

f has a rel min of -30 at $x = 5$ because $f'(x) < 0$ on $(1, 5)$ and $f'(x) > 0$ on $(5, \infty)$.

Find the rel. ext of $f(x) = \frac{x-2}{x+2}$

$f'(x) = \frac{4}{(x+2)^2}$

$f'(x) \neq 0$

$f' \nexists$ at $x = -2$ (but $-2 \notin f$ $\therefore$ not c.n.)

$\therefore$ f has no rel ext because f has no c.n.

Where is f incr/dec?

f is incr on $(-\infty, -2) \cup (-2, \infty)$

because $f'(x) > 0$ on $(-\infty, -2) \cup (-2, \infty)$.

Find the rel ext of $f(x) = \sin 2x$ on $[0, \pi]$.

$f'(x) = 2\cos 2x$

$f' \exists \ \forall x \in [0, \pi]$

$f'(x) = 0 \rightarrow x = \frac{\pi}{4}$ or $x = \frac{3\pi}{4}$

f has a rel max of 1 at $x = \frac{\pi}{4}$
because $f'(x) > 0$ on $(0, \frac{\pi}{4})$
and $f'(x) < 0$ on $(\frac{\pi}{4}, \frac{3\pi}{4})$.

f has a rel min of -1 at $x = \frac{3\pi}{4}$ because
$f'(x) < 0$ on $(\frac{\pi}{4}, \frac{3\pi}{4})$ and $f'(x) > 0$ on $(\frac{3\pi}{4}, \pi)$.

Find the rel ext of $f(x) = \begin{cases} 4-(x+5)^2 & x < -4 \\ 12-(x+1)^2 & x \geq -4 \end{cases}$

$8-10 = -2$
$8-2 = 6$
$12-(3^2)$
$12-9$

$$f'(x) = \begin{cases} -2(x+5) = -2x-10 & x < -4 \\ -2(x+1) = -2x-2 & x > -4 \end{cases}$$

$f'(x) = 0 \rightarrow x = -5$ a $x = -1$
$f' \nexists$ at $x = -4$ because $f'_+(-4) = 6$ but $f'_-(-4) = -2$.

f has rel max of 4 at $x=-5$ because
$f'(x) > 0$ on $(-\infty, -5)$ and $f'(x) < 0$ on $(-5, -4)$.

f has a rel min of 3 at $x = -4$ because
$f'(x) < 0$ on $(-5, -4)$ and $f'(x) > 0$ on $(-4, -1)$

f has rel max of 12 at $x = -1$ because
$f'(x) > 0$ on $(-4, -1)$ and $f'(x) < 0$ on $(-1, \infty)$.