The Mean Value Theorem

But first...

Find the c.n. of $f(x) = \begin{cases} 3x-6 & x \overset{\le}{\ge} 3 \\ x^2 & x \overset{>}{<} 3 \end{cases}$

$f'(x) = \begin{cases} 3 & x \overset{<}{>} 3 \\ 2x & x \overset{>}{<} 3 \end{cases}$

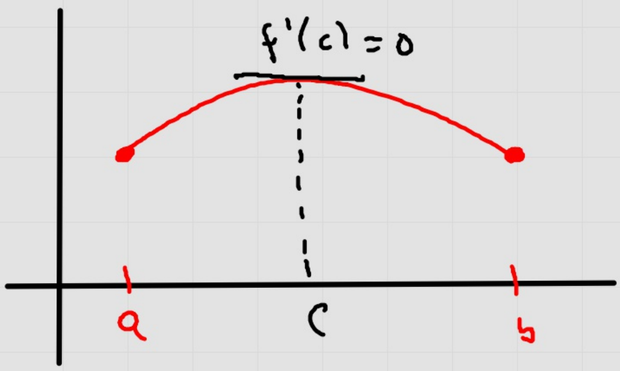
$f'(x) = 0 \Rightarrow x = 0$

f' $\nexists$ at $x=3$ because $f'_+(3) = 3$ but $f'_-(3) = 6$. ✓

$\therefore$ c.n. are $x=0$ and $x=3$.

Rolle's Thm

If f is cont on $[a,b]$ and
f is diff on (a,b) and
$f(a)=f(b)$ then $\exists$ a $c \in (a,b)$
such that $f'(c)=0$.



Given $f(x) = x^3 + x^2 - 2x + 1$ on $[-2, 0]$. Verify that R.T. holds for f on $[-2,0]$ and then find the c gauranteed by the theorem.

f is cont on $[-2,0]$ and diff on $(-2,0)$ and $f(-2) = f(0) = 1$ $\therefore$ RT holds.

$f'(x) = 3x^2 + 2x - 2$

$f'(x) = 0 \rightarrow x = -1.215$ or $x = .549$

Since $.549 \notin (-2,0)$, $c = -1.215$ only.

Mean Value Thm

If f is cont on $[a,b]$ and diff on (a,b)

then $\exists\, c \in (a,b)$ such that

$$\underbrace{f'(c)}_{\text{inst rate of change in fun. vals.}} = \underbrace{\frac{f(b)-f(a)}{b-a}}_{\text{avg. rate of change in fun. val}}.$$

Given $f(x) = 2x^3 + x^2 - x - 1$ on $[0,2]$. Verify that the MVT holds for f on $[0,2]$ and find the c that satisfies the cond. of the MVT.

f is cont on $[0,2]$ and f is diff on $(0,2)$

$\therefore$ MVT holds.

$f'(x) = 6x^2 + 2x - 1$

$$6x^2 + 2x - 1 = \frac{f(2) - f(0)}{2 - 0}$$

$$6x^2 + 2x - 1 = 9 \rightarrow x = -1.468 \text{ or } x = 1.135$$

Since $-1.468 \notin (0,2)$, $c = 1.135$ only.