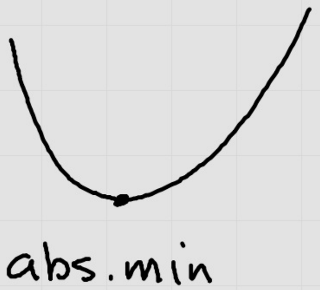
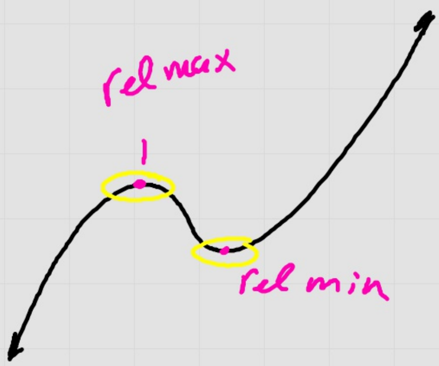
Extrema

Relative and Absolute Maximums

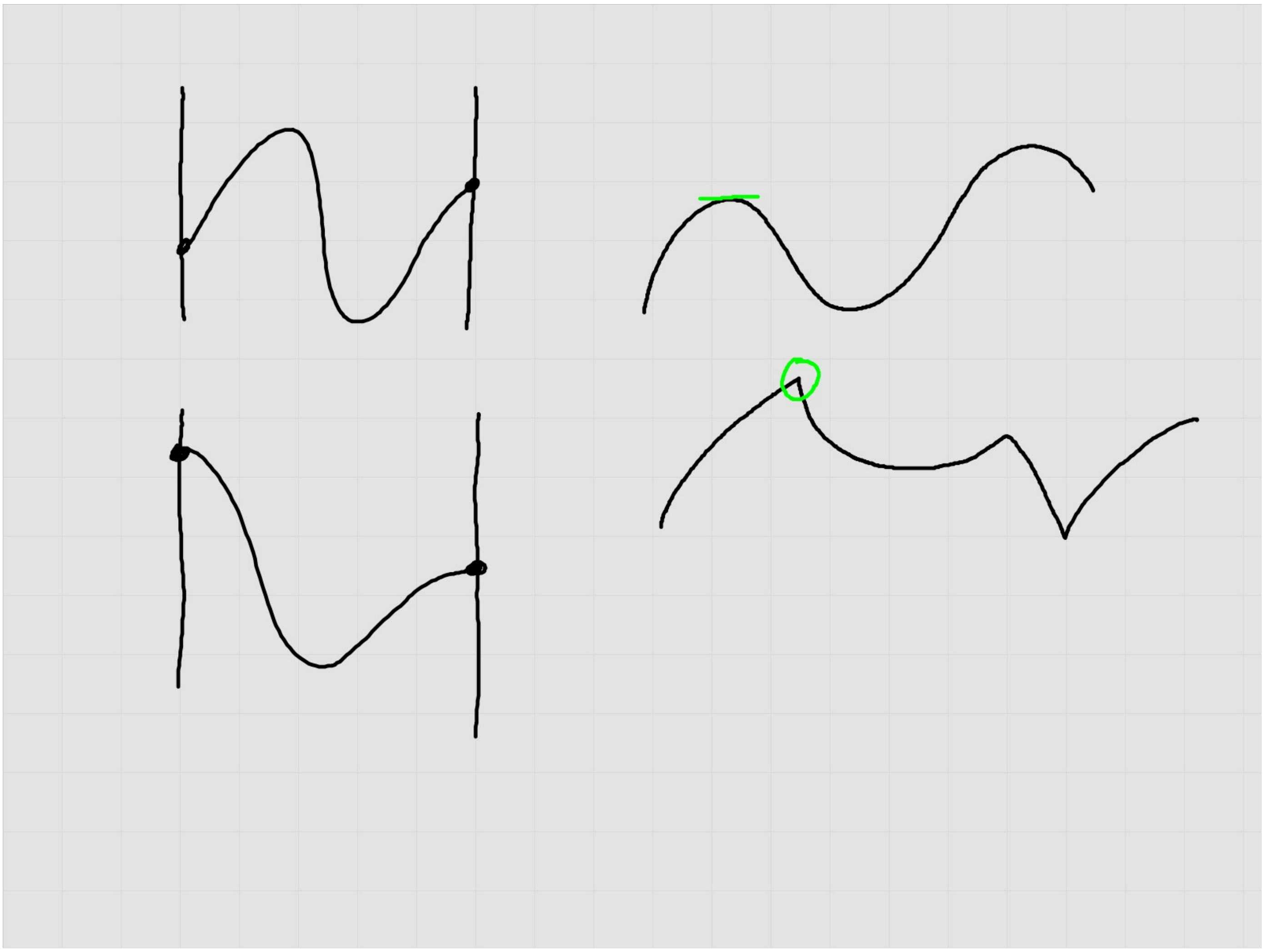
local





Absolute can occur at endpoints

Rel ext must occur on some interval



Values of x where $f'(x)=0$ or $f'(x)\nexists$ are called __critical numbers__. (must be in f)

★ If f has rel. ext. they MUST occur at c.n. ★

BUT just because f has c.n. does not ness. mean has rel ext. ★

$f(x) = x^3$

$f'(x) = 3x^2$

$f'\ \exists\ \forall x$

$f'(x)=0 \rightarrow x=0$

$\therefore\ x=0$ is c.n.

$$f(x) = \frac{3}{x-5}$$

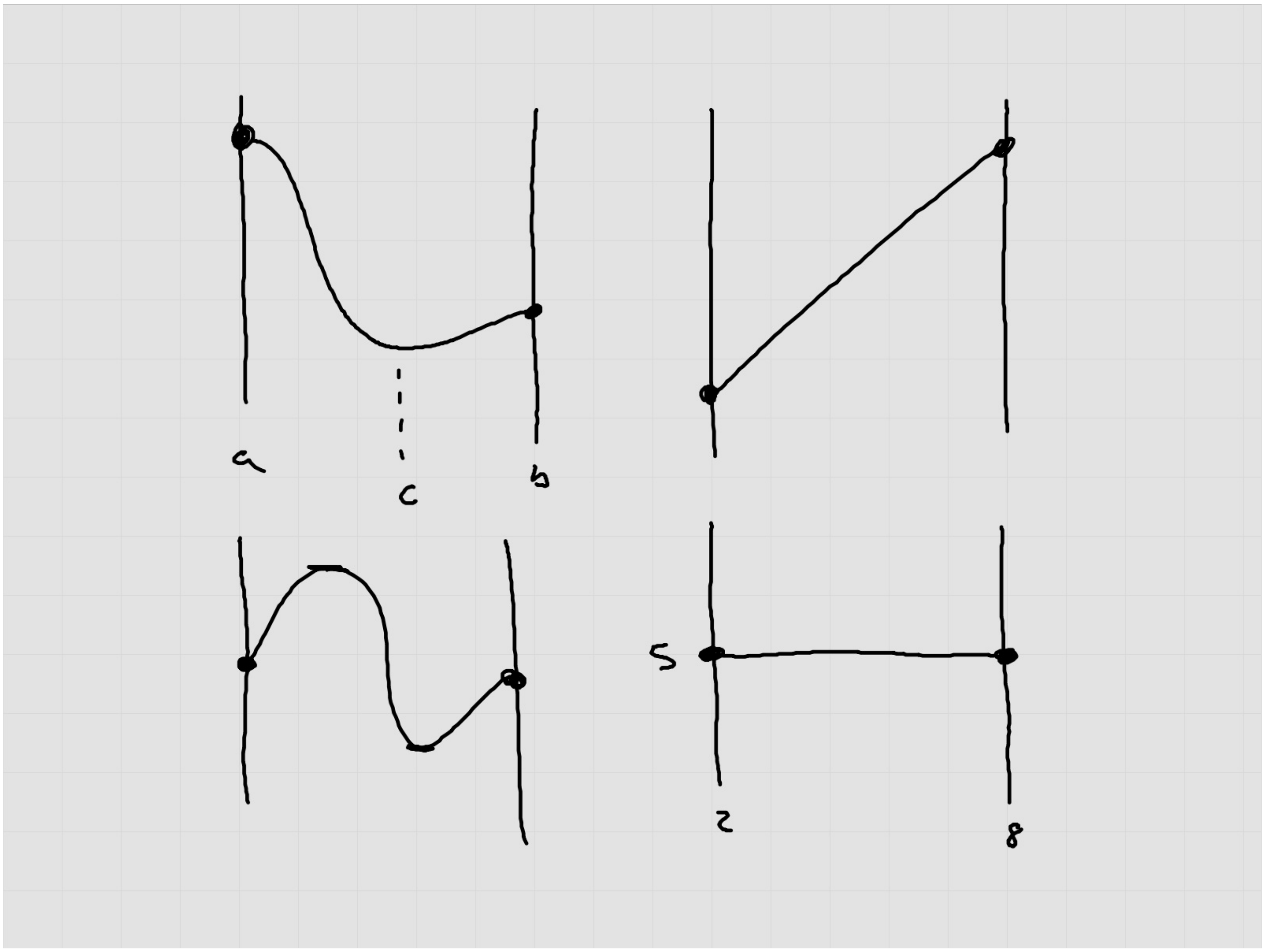
$$f'(x) = -3(x-5)^{-2}(1)$$

$$= -\frac{3}{(x-5)^2}$$

$f'(x) \neq 0$

$f'(x) \nexists$ at $x=5$

$5 \notin f$ $\therefore$ $x=5$ not a c.n.


a
c
b
5
2
8

Extreme Value Theorem

If f is cont on $[a,b]$ then f has an abs max func. val AND an abs. min func. value.

To find abs ext on $[a,b]$

1. find f' & c.n.
2. f(endpts)
   f(c.n.)
3. Big val - abs max
   Small val - abs min.

Find abs. ext. of $f(x) = 4x^3 - 15x^2 + 12x$ $[0,3]$.

$f'(x) = 12x^2 - 30x + 12$

$f' \ \exists \ \forall x$

$f'(x) = 0 \longrightarrow x = \frac{1}{2}$ or $x = 2$

$f(0) = 0$

$f(3) = 9$

$f(\frac{1}{2}) = \frac{11}{4}$

$f(2) = -4$

$\therefore$ Abs max is 9 at $x=3$

Abs min is -4 at $x=2$.