

Derivatives of the Inverse Trigonometric Functions

$$y = \sin^{-1} x$$

$$x = \sin y$$

$$1 = \cos y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\cos y = \sqrt{1 - \sin^2 y}$$
$$= \sqrt{1 - x^2}$$

$$D_x [\sin^{-1} f(x)] = \frac{f'(x)}{\sqrt{1 - f(x)^2}}$$

$$f(x) = \sin^{-1} e^x$$

$$f'(x) = \frac{e^x}{\sqrt{1 - e^{2x}}}$$

$$y = \cos^{-1} x$$

$$x = \cos y$$

$$1 = -\sin y \frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{1}{\sin y}$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\begin{aligned}\sin y &= \sqrt{1 - \cos^2 y} \\ &= \sqrt{1 - x^2}\end{aligned}$$

$$D_x [\cos^{-1} f(x)] = -\frac{f'(x)}{\sqrt{1-f(x)^2}}$$

$$y = \tan^{-1} x$$

$$x = \tan y$$

$$1 = \sec^2 y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\tan^2 y + 1 = \sec^2 y$$

$$x^2 + 1 = \sec^2 y$$

$$D_x [\tan^{-1} f(x)] = \frac{f'(x)}{1 + f(x)^2}$$

$$D_x [\cot^{-1} f(x)] = - \frac{f'(x)}{1 + f(x)^2}$$

$$y = \sec^{-1} x$$

$$x = \sec y$$

$$1 = \sec y \tan y \frac{dy}{dx}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sec y \tan y} \\ &= \frac{1}{x \tan y} \\ &= \frac{1}{x \sqrt{x^2 - 1}} \end{aligned}$$

$$1 + \tan^2 y = \sec^2 y$$

$$\begin{aligned} \tan y &= \sqrt{\sec^2 y - 1} \\ &= \sqrt{x^2 - 1} \end{aligned}$$

$$D_x [\sec^{-1} f(x)] = \frac{f'(x)}{f(x) \sqrt{f(x)^2 - 1}}$$

$$D_x [\csc^{-1} f(x)] = - \frac{f'(x)}{f(x) \sqrt{f(x)^2 - 1}}$$

$$f(x) = \sin^{-1} \sqrt{x}$$

$$\begin{aligned} f'(x) &= \frac{\frac{1}{2\sqrt{x}}}{\sqrt{1-(\sqrt{x})^2}} \\ &= \frac{1}{2\sqrt{x}\sqrt{1-x}} \\ &= \frac{1}{2\sqrt{x-x^2}} \end{aligned}$$

$$g(x) = \tan^{-1}(3x+5)$$

$$g'(x) = \frac{3}{1+(3x+5)^2}$$

$$f(x) = e^{-x} \sin^{-1}(e^{-x})$$

$$\sqrt{f'(x)} = (e^{-x}) \frac{-e^{-x}}{\sqrt{1-(e^{-x})^2}} + (\sin^{-1}(e^{-x}))(-e^{-x})$$

$$= \frac{-\sqrt{e^{2x}-1} - e^{2x} \sin^{-1}(e^{-x})}{e^{3x}}$$

$$\sqrt{1 - \frac{1}{e^{2x}}}$$

$$\sqrt{\frac{e^{2x}-1}{e^{2x}}}$$

$$\left(-\frac{\sqrt{e^{2x}-1}}{e^{3x}} \right)$$

$$- \frac{\sin^{-1}(e^{-x})}{e^x}$$

$$h(x) = \ln [\tan^{-1} x^2]$$

$$h'(x) = \frac{\frac{2x}{1+x^4}}{\tan^{-1} x^2} \cdot \frac{2x}{1+x^4}$$

$$f(x) = \sin^{-1}(\ln x)$$

$$f'(x) = \frac{\frac{1}{x}}{\sqrt{1-\ln^2 x}}$$