Derivative of Logarithmic Functions

(and the General Exponential Function)

$f(x) = \log_a x$

$y = \log_a x \iff a^y = x$

$\log_a a^x = x$

$a^{\log_a x} = x$

$f(x) = \ln(x)$

$y = \ln x \iff e^y = x$

$\ln e^x = x$

$e^{\ln x} = x$

$\ln e = 1$

$\ln 1 = 0$

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^n = n \log_a x$$

$$y = \log_a x$$

$$a^y = x$$

$$\ln a^y = \ln x$$

$$y \ln a = \ln x$$

$$y = \frac{\ln x}{\ln a}$$

$$\boxed{\log_a x = \frac{\ln x}{\ln a}}$$

Change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$\log_9 15 = \frac{\log_5 15}{\log_5 9}$$

$$\log_9 15 = \frac{\ln 15}{\ln 9}$$

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$16 \cdot 8 = 1\underline{28}$

$4 + 3 = 7$

$$2^{3^x} = 5$$

$$\ln 2^{3^x} = \ln 5$$

$$3^x = \frac{\ln 5}{\ln 2}$$

$$\ln 3^x = \ln\left(\frac{\ln 5}{\ln 2}\right)$$

$$x \ln 3 = \ln\left(\frac{\ln 5}{\ln 2}\right)$$

$$x = \frac{\ln\left(\frac{\ln 5}{\ln 2}\right)}{\ln 3}$$

$$2^{3^x} = 5$$

$$\log_2 2^{3x} = \log_2 5$$

$$3^x = \log_2 5$$

$$\log_3 3^x = \log_3(\log_2 5)$$

$$x = \log_3(\log_2 5)$$

$$y = \ln x$$

$$e^y = x$$

$$e^y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y}$$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$D_x[\ln x] = \frac{1}{x}$$

$$\boxed{D_x[\ln f(x)] = \frac{f'(x)}{f(x)}}$$

$$f(x) = \ln(x^2 + 5x)$$

$$f'(x) = \frac{2x+5}{x^2+5x}$$

$$y = \log_a x$$

$$D_x[\log_a f(x)] = \frac{f'(x)}{f(x)\ln a}$$

$$\log_a x = \frac{\ln x}{\ln a}$$

$$D_x[\log_a x] = D_x\left[\frac{\ln x}{\ln a}\right]$$

$$= \frac{1}{\ln a} D_x[\ln x]$$

$$= \frac{1}{x \ln a}$$

$$D_x(\log_a x) = \frac{1}{x \ln a}$$

$$\boxed{D_x\left[\log_a f(x)\right] = \frac{f'(x)}{f(x)\ln a} = \frac{f'(x)\log_a e}{f(x)}}$$

$$\log_a x = \frac{\ln x}{\ln a}$$

For $x = e$

$$\log_a e = \frac{\ln e}{\ln a}$$

$$\log_a e = \frac{1}{\ln a}$$

$$\boxed{f(x) = \log_5(3x-1)}$$

$$f'(x) = \frac{3}{(3x-1)\ln 5}$$

$$= \frac{3\log_5 e}{(3x-1)}$$

$$y = a^x$$

$$D_x\left[a^x\right] = a^x \ln a$$

$$a^x = e^{\ln a^x}$$

$$\boxed{D_x\left[a^{f(x)}\right] = a^{f(x)} f'(x) \ln a}$$

$$a^x = e^{x \ln a}$$

$$f(x) = 8^{x^2}$$

$$D_x\left[a^x\right] = D_x\left[e^{x \ln a}\right]$$

$$f'(x) = \left(8^{x^2}\right)(2x)(\ln 8)$$

$$= e^{x \ln a} \ln a$$

$$= e^{\ln a^x} \ln a$$

$$= a^x \ln a$$

$$f(x) = 5^{\sin x^2}$$

$$f'(x) = \left(5^{\sin x^2}\right)\left(2x\cos x^2\right)\left(\ln 5\right)$$

$$g(x) = e^{\sin x^2}$$

$$g'(x) = e^{\sin x^2}\, 2x\cos x^2$$

$$D_x\left[e^{f(x)}\right] = e^{f(x)} f'(x)$$

$$D_x\left[a^{f(x)}\right] = a^{f(x)} f'(x) \ln a$$

$$D_x\left[\ln f(x)\right] = \frac{f'(x)}{f(x)}$$

$$D_x\left[\log_a f(x)\right] = \frac{f'(x)}{f(x)\ln a} = \frac{f'(x)\log_a e}{f(x)}$$

$\text{fun}^{\#}$ $\#^{\text{func.}}$

"logarithmic differentiation"

$\longrightarrow \quad y = x^{x^3}$

$\ln y = \ln x^{x^3}$

$\ln y = x^3 \ln x$

$\frac{1}{y}\frac{dy}{dx} = \frac{x^3}{x} + 3x^2 \ln x$

$\frac{dy}{dx} = y\left(x^2 + 3x^2 \ln x\right)$

$\frac{dy}{dx} = x^{x^3}\left(x^2 + 3x^2 \ln x\right)$

$$y = \frac{5x-2}{3x+1}$$

$$\ln y = \ln(5x-2) - \ln(3x+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{5}{5x-2} - \frac{3}{3x+1}$$

$$\frac{dy}{dx} = \frac{5x-2}{3x+1}\left(\frac{5}{5x-2} - \frac{3}{3x+1}\right)$$

$$y = \sqrt[3]{x^5 - 6}$$

$$\ln y = \frac{1}{3} \ln(x^5 - 6)$$

$$\frac{dy}{dx} = \frac{1}{3} \sqrt[3]{x^5 - 6} \left(\frac{5x^4}{x^5 - 6} \right)$$